UNCLASSIFIED

AD NUMBER	
AD041742	
CLASSIFICATION CHANGES	
TO:	**unclassified**
FROM:	**confidential**
LIMITATION CHANGES	
TO:	**Approved for public release; distribution is unlimited.**
FROM:	**Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; JUN 1953. Other requests shall be referred to Bureau of Ordnance, Washington, DC.**
AUTHORITY	
30 Jun 1965, DoDD 5200.10; BUORD ltr dtd 4 Apr 1967	

THIS PAGE IS UNCLASSIFIED

UNCLASSIFIED

AD ________

Reproduced by the

ARMED SERVICES TECHNICAL INFORMATION AGENCY
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA

DOWNGRADED AT 3 YEAR INTERVALS:
DECLASSIFIED AFTER 12 YEARS
DOD DIR 5200.10

UNCLASSIFIED

Armed Services Technical Information Agency

Because of our limited supply, you are requested to return this copy WHEN IT HAS SERVED YOUR PURPOSE so that it may be made available to other requesters. Your cooperation will be appreciated.

AD 41742

NOTICE: WHEN GOVERNMENT OR OTHER DRAWINGS, SPECIFICATIONS OR OTHER DATA ARE USED FOR ANY PURPOSE OTHER THAN IN CONNECTION WITH A DEFINITELY RELATED GOVERNMENT PROCUREMENT OPERATION, THE U. S. GOVERNMENT THEREBY INCURS NO RESPONSIBILITY, NOR ANY OBLIGATION WHATSOEVER; AND THE FACT THAT THE GOVERNMENT MAY HAVE FORMULATED, FURNISHED, OR IN ANY WAY SUPPLIED THE SAID DRAWINGS, SPECIFICATIONS, OR OTHER DATA IS NOT TO BE REGARDED BY IMPLICATION OR OTHERWISE AS IN ANY MANNER LICENSING THE HOLDER OR ANY OTHER PERSON OR CORPORATION, OR CONVEYING ANY RIGHTS OR PERMISSION TO MANUFACTURE, USE OR SELL ANY PATENTED INVENTION THAT MAY IN ANY WAY BE RELATED THERETO.

Reproduced by

DOCUMENT SERVICE CENTER

KNOTT BUILDING, DAYTON, 2, OHIO

CONFIDENTIAL

AD No. 41762
ASTIA FILE COPY

RAMJET TECHNOLOGY

CONFIDENTIAL

APL/JHU TG 154-13
Copy No. 37

Chapter 13
EXIT-NOZZLE DESIGN

by

M. L. STREIFF

Consolidated Vultee
Aircraft Corporation

Published by

THE JOHNS HOPKINS UNIVERSITY
APPLIED PHYSICS LABORATORY
Silver Spring Maryland

Operating under Contract NOrd 7386 with
the Bureau of Ordnance, Department of the Navy

CONFIDENTIAL

This document contains information affecting the national defense of the United States within the meaning of the Espionage Laws, Title 18, USC, Section 793 and 794. Its transmission or revelation of its contents in any manner to an unauthorized person is prohibited by law.

OCT 6 - 1954

54AA 60586

CHAPTER 13

EXIT-NOZZLE DESIGN

by
M. L. Streiff
Consolidated Vultee Aircraft Corporation

(Manuscript submitted by author June 1953)

54AA 60586

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

TABLE OF CONTENTS

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

13. EXIT-NOZZLE DESIGN

by

M. L. Streiff

13.1 RELATION OF EXIT NOZZLE TO COMPLETE RAMJET

Effect of Exit Constriction on Powerplant Performance

Since the main purpose of early ramjet models was to obtain a maximum amount of thrust without very much concern over efficiency, a straight tailpipe with no combustion-chamber exit nozzle was often used. As techniques developed, the efficiency with which thrust is produced became of paramount concern. Variations of the combustion-chamber exit-nozzle geometry make the efficient production of thrust possible by means of changes to the powerplant geometry or the character of the internal flow (see Chapter 2).

For example, consider a ramjet that is designed to fly at Mach number 1.50 and which, for simplicity, is assumed to have a normal shock inlet and sonic exit. In terms of the maximum area, the exit and inlet areas are to be variable so that shock on rim conditions may always be maintained at the diffuser inlet. By using Eqs. (13.1-1), (13.1-2), and (13.1-3), solutions of the available thrust coefficient, C_t, the inlet area ratio, A_o/A_{max}, and the fuel specific impulse, I_f, were obtained in terms of the constriction ratio A_c/A_b, and the combustor heat release, T_{t_o}/T_{t_c}.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

$$C_t = \frac{F_{net}}{q_o\ A_{max}} = \frac{F_c - F_o - P_o\ (A_c - A_o)}{q_o\ A_{max}}$$

$$= \left(\frac{P_t}{q}\right)_o \frac{A_c}{A_{max}} \left\{ \left(\frac{P_{t_c}}{P_{t_o}}\right)\left(\frac{f}{P_t}\right)_c - \left(\frac{P}{P_t}\right)_o - \left[\frac{P_{t_c}}{P_{t_o}}\ \frac{(\overset{\circ}{Pm}/P_t)_c}{(\overset{\circ}{Pm}/P_t)_o} \sqrt{\frac{T_{t_o}}{T_{t_c}}}\right]\right.$$

$$\left. \left[(f/P_t)_o - (P/P_t)_o\right] \right\} \qquad (13.1\text{-}1)$$

$$\frac{A_o}{A_{max}} = \frac{A_c}{A_b}\ \frac{A_o}{A_c} = \left(\frac{A_c}{A_b}\right)\left(\frac{P_{t_c}}{P_{t_o}}\right) \frac{(\overset{\circ}{Pm}/P_t)_c}{(\overset{\circ}{Pm}/P_t)_o} \sqrt{\frac{T_{t_o}}{T_{t_c}}} \qquad (13.1\text{-}2)$$

$$I_f = \frac{F_{net}}{W_f} = \frac{C_t\ q_o\ A_{max}}{W_{f_1}/\eta} = \frac{C_t\ q_o\ A_{max}}{W_a/(a/f)(\eta)}$$

$$= C_t \left((q/P)_o\ \frac{\sqrt{(T_t/T)_o}}{\overset{\circ}{m}_o}\right) \frac{a/f\ \eta_c\ \sqrt{T_o}}{(A_o/A_{max}} \qquad (13.1\text{-}3)$$

The assumptions and results shown in Fig. 13.1-1 indicate that a given thrust coefficient can be obtained with a variety of constriction ratio and combustor heat release combinations. The important point is that some combinations give a higher I_f, that is, the same thrust is obtained with less fuel consumption. With the assumed conditions and a straight tailpipe ($A_c/A_b = 1.0$) no optimum design is possible for a given C_t requirement. In Fig. 13.1-1 the points of maximum I_f are connected and the resulting curve is labeled "maximum efficiency". The increase of the maximum I_f over that for a straight tailpipe varies with C_t.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

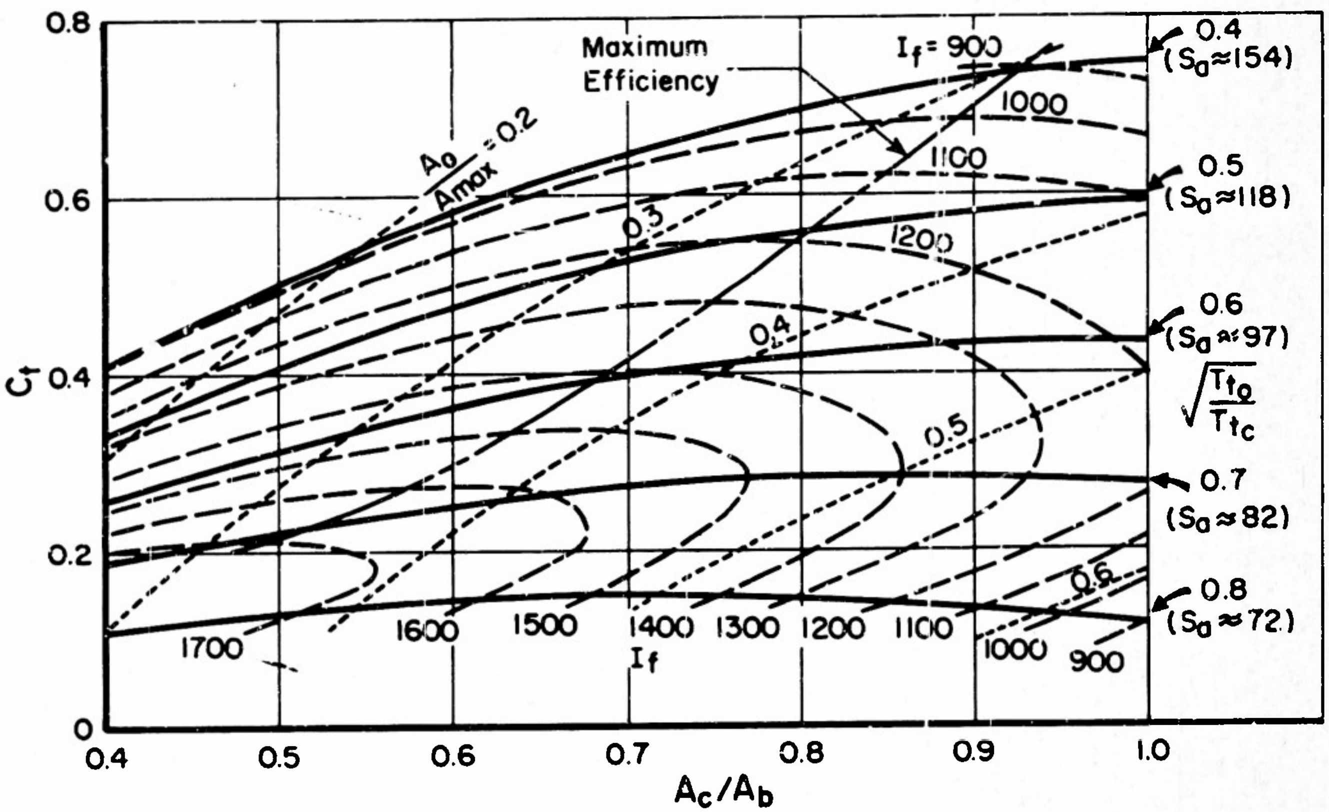


Fig. 13.1-1 **EXAMPLE OF EFFICIENCY GAINS USING AN EXIT CONSTRICTION-- NO BOATTAIL DRAG**

These curves were obtained under the following conditions: $M_o = 1.50$, normal shock inlet, sonic exit, shock on rim, $P_{t_2}/P_{t_o} = 1 - 0.3\,(q/P_t)_o$ (after shock), $P_{t_c}/P_{t_2} = f(T_{t_o}/T_{t_c}$ and $A_c/A_b)$ $(C_{d_b} = 1.0)$, $A_b = A_{max}$, $T_o = 393^oR$ ($Z > 35{,}000$ ft), and $\eta = 80$ per cent.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

For high air specific impulse and high thrust coefficient, the increase in I_f is small, while for lean burning conditions at a lower C_t the increase is almost 100 per cent. While the values obtained apply only to the particular conditions chosen, a maximum efficiency similarly exists for other conditions.

Maximum I_f, although important, does not measure the over-all performance. In the example, where $C_t = 0.7$, there is little change in I_f with constriction ratio, although the inlet area is smaller as the constriction ratio is decreased. The smaller inlet would allow more volume for packaging or storage of fuel and subsequent increase in missile range. Furthermore, the variations of inlet and exit geometry give rise to drag changes which can completely alter the design. The drag on the outside of the inlet would change very little with A_o/A_{max} for an external cone at the inlet having, for example, a 10-degree total angle. The change in drag resulting from reduction in exit diameter may be very important.

In the discussion thus far the flow in the exit nozzle has been assumed to be isentropic. Real flows are known to deviate from "ideal" one-dimensional isentropic conditions either by loss of total pressure or by variation of the stream properties in a plane normal to the mean flow direction. The effect of these deviations on the equations for thrust coefficient and fuel specific impulse will now be considered. Although total pressure loss and stream property variations are interrelated, their effects on C_t and I_f will be discussed separately.

Equation (13.1-1), which is derived from momentum and continuity relations, shows that, if there were a loss of total pressure, the thrust-coefficient would be reduced since the only term in Eq. (13.1-1) which is not affected by a loss (expressible in terms of P_{t_c}/P_{t_o}) is the subtractive term, $(P/P_t)_o$ It can be shown that the percentage loss in C_t would

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

exceed the percentage loss of total pressure. The airflow (expressed as A_o/A_{max}) in Eqs. (13.1-2) and (13.1-3) would be reduced by a percentage equal to the percentage loss of total pressure. Consequently I_f would be reduced.

In the previous example, a discharge coefficient (C_d) of 0.97, caused entirely by loss of total pressure, would have reduced C_t by about six to twenty per cent, depending on C_t, and would reduce I_f by about three to seventeen per cent. The large percentage changes occur at the low values of C_t, emphasizing the importance of reducing nozzle losses in low-impulse, high-efficiency powerplants.

Although unidimensional relations are normally used for a majority of ramjet calculations, there are profiles of pressure, velocity, and total temperature at any station plane. The velocity and pressure profiles are most pronounced at changes in cross-sectional area, such as the exit nozzle. The total temperature profiles are a function of the combustor geometry and mixing. Profiles of velocity and pressure do not necessarily arise from wall friction or other total pressure losses. These profiles can occur in an isentropic flow near a sonic throat or in a supersonic region. The effect of these profiles on discharge or thrust is therefore completely independent of any total pressure losses that may be present.

The most accurate estimate of nozzle performance may best be made by assuming that unidimentional equations may be validly applied to a small portion of the stream and by integrating the desired stream properties for a known or an assumed profile. For example, assume the Mach number across an axially-symmetric exit constriction varies parabolically from a Mach number of 0.8 at the centerline to 1.2 at the wall (these values might be typical for a relatively short nozzle inlet resulting from small radii of wall curvature). Assuming also that total pressure and temperature are uniform across

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

the section, discharge becomes a function of the integrated value of $P\mathring{m}/P_t$; stream thrust with this reduced discharge becomes a function of the integration of f/P_t and the exit stream thrust per unit flow is a function of the ϕM integration. Integrated values for the assumed examples are:

$$(P\mathring{m}/P_t)avg = 0.9885\ (P\mathring{m}/P_t)\ M = 1$$

$$(f/P_t)avg = 0.9932\ (f/P_t)\ M = 1$$

$$(\phi M)\ avg = 1.0047\ (\phi M)\ M = 1.$$

Thus, by Eq. (13.1-2), the airflow would be 0.9885 as great as with a uniform Mach number equal to one. The effect on C_t is not obvious [Eq. (13.1-1)] since $(f/P_t)_c$ and $(P\mathring{m}/P_t)_c$ both decrease. The exit Mach number variation was applied to the previous ramjet example with the result that C_t was found to decrease by an amount approximately equal to the decrease in airflow. Consequently I_f did not change for this example even for the low values of C_t. However, this variation of C_t and I_f is not necessarily typical, since the variation depends on the relative values in the C_t equation. One would expect the variations C_t and I_f to be greater with a supersonic exit than with the sonic exit.

Since no general statement can be made regarding the effects of exit-flow profiles, the best means of determining their effects is by integration of the particular case using test data or the method of characteristics for determination of the profiles. It is probable, however, that the effect of exit-flow profiles on C_t or I_f will be small except possibly for very high impulse missiles.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

Boattail Drag Effects with Exit Constriction

The reduction of the outside diameter toward the exit of a ramjet is commonly referred to as a boattail which has pressures below ambient over its surface and thus contributes a drag. Since the boattail and the exit nozzle have the exit area in common, the configuration at the exit must consider both these components in order to obtain a true optimum.

To illustrate the importance of the effect of boattail drag, the original example was extended by defining a net thrust coefficient, C_t net, which includes not only the powerplant thrust coefficient, C_t, but the drag increment, $\Delta C_{t_{bt}}$, resulting from the boattail. It was assumed that the pressure coefficient of the boattailed area was -0.20 of the free-stream dynamic pressure. The boattail was assumed to extend from the maximum diameter to the diameter of the sonic exit.

The following equations were used to obtain the "net" thrust coefficient and fuel specific impulse. Equation (13.1-5) gives reasonably good agreement with the boattail pressure coefficients of Ref. [1].

$$\Delta C_{t\,(net)} = C_t - \Delta C_{t_{bt}} \qquad (13.1\text{-}4)$$

$$\Delta C_{t_{bt}} = \frac{D_{bt}}{q_o\, A_{max}} = \frac{\Delta P}{q_o}\left(1 - \frac{A_c}{A_{max}}\right) \qquad (13.1\text{-}5)$$

$$I_{f\,(net)} = I_f \;\frac{C_{t\,(net)}}{C_t}. \qquad (13.1\text{-}6)$$

The results are shown in Fig. 13.1-2 for the same assumptions as in Fig. 13.1-1 except for the inclusion of the boattail drag effects. Although the values at A_c/A_b = 1.0 are identical for these two figures, the available thrust at all other values

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

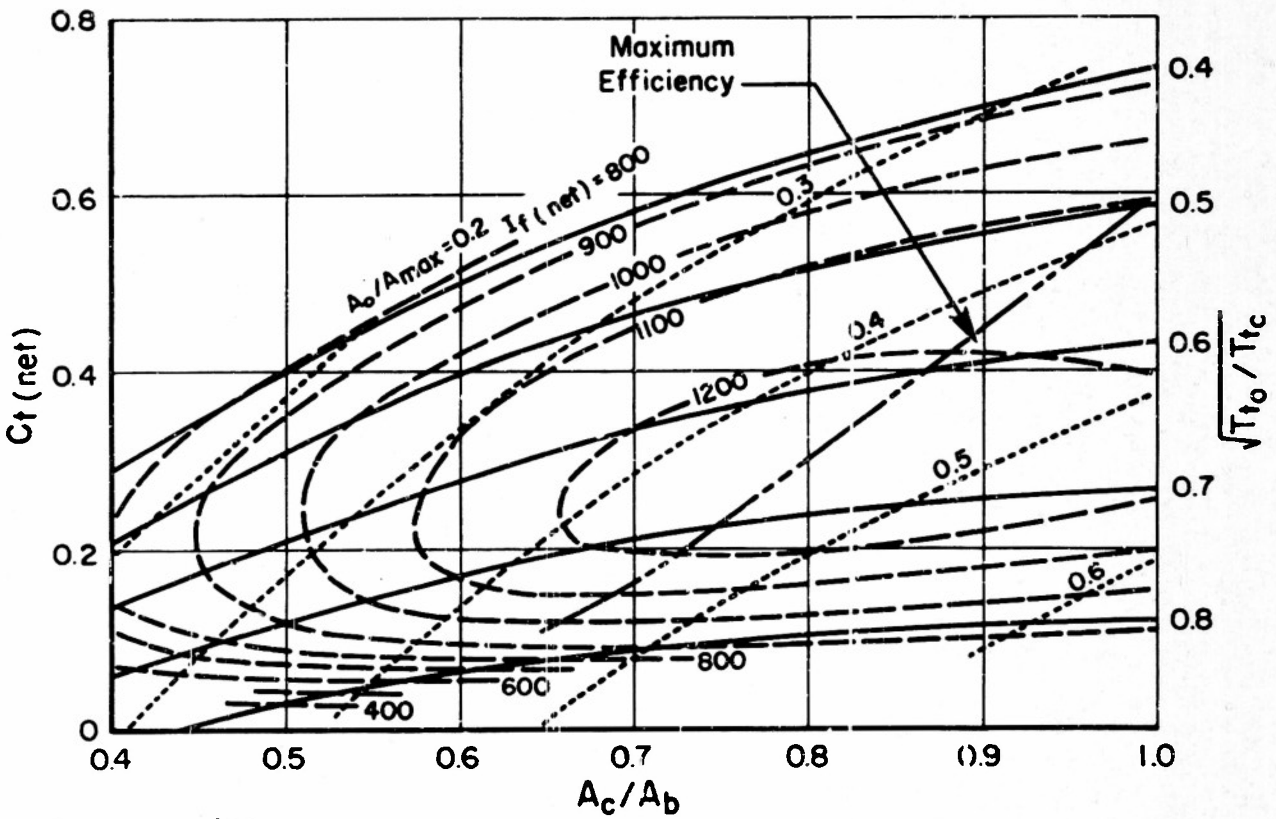


Fig. 13.1-2 **EXAMPLE SHOWING THE EFFECT OF BOATTAIL DRAG ON EFFICIENCY GAINS USING AN EXIT CONSTRICTION**

These curves were obtained under conditions similar to Fig. 13.1-1 except for boattail drag. $\Delta C_{t_{bt}} = \frac{\Delta P}{q_o}\left(1 - \frac{A}{A_{max}}\right)$ $\Delta P/q_o = -0.20.$

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

of A_c/A_b is reduced. The most important difference is the change of the entire pattern of the I_f or efficiency lines. While there is still an optimum constriction ratio for maximum I_f, the difference in I_f for a change in A_c/A_b is considerably less than in Fig. 13.1-1. The boattail drag has completely nullified the high values of I_f in the low range of Fig. 13.1-1.

It is not uncommon for powerplant designers to consider only the momentum change of the internal flow and the atmospheric force on the difference in exit and inlet area for definition of thrust coefficient available from the powerplant. The above example clearly demonstrates the futility of trying to accomplish an optimum without consideration of the effects on other components external to the powerplant.

The Effect of an Exit Divergence on Powerplant Performance

The De Laval-type exit nozzle consists of a constriction followed by a divergent section. For the range of pressures commonly encountered in ramjets there is sufficient pressure ratio across the exit nozzle to maintain supersonic flow in the divergent section.

Since the static pressure at the throat is usually considerably above ambient, an expansion to ambient pressure in the diverging section results in a positive differential pressure with respect to free-stream pressure and thrust. The differential pressure with respect to the pressure which would have existed on a boattail is even greater.

One means of expressing the thrust increase is by means of ϕM, the ratio of the stream thrust at any Mach number, M, to the stream thrust at $M = 1.0$. The value of ϕM is, of course, unity at $M = 1.0$ and rises as M increases (or decreases). The increase in the thrust coefficient over that of a sonic exit

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

can be expressed by the following equation

$$\Delta C_t = \frac{(F_e - F_c) - P_o \, (A_e - A_c)}{q_o \, A_{max}} \, . \qquad (13.1\text{-}7)$$

Using the air specific impulse, S_a, and the stream thrust ratio, C_{f_e}, which represents the ratio of actual to ideal stream thrust, Eq. (13.1-7) becomes

$$\Delta C_t = \frac{W_a \, S_a \, (C_{f_e} \, \phi M_e - 1.0) - P_o \, (A_e - A_c)}{q_o \, A_{max}} \qquad (13.1\text{-}8)$$

It is necessary, as shown in the previous section, to include boattail drag effects. Including these in Eq. (13.1-8) results in the following equation

$$C_{t(net)} = \frac{W_a \, S_a \, (C_{f_e} \, \phi M_e - 1.0) - P_o \, (A_e - A_c)}{q_o \, A_{max}} + \frac{\Delta P}{q_o} \, \frac{A_{max} - A_e}{A_{max}}$$

$$- \frac{\Delta P}{q_o} \left(\frac{A_{max} - A_c}{A_{max}} \right) \qquad (13.1\text{-}9)$$

where $\Delta P/q_o$ is negative.

Equation (13.1-9) was solved for the previous example assuming an expansion to the free-stream pressure. No calculations were made for $A_c/A_b > 0.9$, since beyond this value the exit area would have exceeded the maximum area defined as equal to A_b. Calculations indicate that, for $C_{f_e} = 1.00$, C_t would increase about 0.02 to 0.03 over the entire range of Fig. 13.1-2 (except for $A_c/A_b > 0.9$). However, if a value of $C_{f_e} = 0.97$ were assumed, C_t would be decreased about 0.01 to 0.02.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

Use of Eq. (13.1-9) for assumed expansion to the base pressure shows that, for $C_{f_e} = 1.0$, C_t would increase 0.03 to 0.04 while, for $C_{f_e} = 0.97$, C_t would decrease 0.01 to 0.02. The slight improvement over expansion to P_o results from the larger exit, thus reducing the boattail drag. These small changes in C_t from the values in Fig. 13.1-2 (for a sonic nozzle) would cause little percentage change in C_t or I_f at the high values of C_t, perhaps five per cent. At the lower values of C_t, however, the percentage changes in C_t and I_f become appreciable. This again demonstrates the need for high nozzle thrust efficiency for ramjet powerplants having a low C_t. It was found in the example that the maximum thrust was not obtained with expansion to ambient pressure. Considering the exit nozzle alone this would seem to be ideal since thrust is obtained as a result of the positive pressure existing on the diverging wall relative to free-stream static pressure. Where a boattail is involved, it is the pressure relative to that on the boattailed area which is important. Further expansion of the exit nozzle, at least to this boattail pressure, appears desirable.

Since the air and fuel flows are unaffected by the divergent section (with sonic throat velocities), any increase in C_t from this section will result in an increase in I_f.

The optimum would be that condition which gives the highest C_t increment considering the nozzle exit-stream thrust, the nozzle losses, and the boattail drag.

The variation of the thrust increment of a diverging nozzle compared to a sonic nozzle varies with flight Mach number. The general trend is to provide a higher C_t relative to a sonic nozzle as the flight Mach number increases. Increase of the flight Mach number causes the pressure ratio, P_{t_b}/P_e, to increase, permitting a higher value for ϕM_e. In addition,

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

the relative effect of static pressure forces, e.g., $P_o\,(A_e - A_o)$ and boattail drag diminish as Mach number increases.

To demonstrate the order of magnitude of the Mach number trend, the following equation was evolved for the C_t increase of a supersonic or divergent exit nozzle over the C_t of a sonic exit nozzle. It is equivalent to Eq. (13.1-9) but of slightly different form which may be of value for computational purposes.

$$\Delta C_t = \frac{(F_e - F_c) - P_o\,(A_e - A_c)}{q_o\,A_{max}} + \frac{\Delta P}{q_o}\left(\frac{A_{max} - A_e}{A_{max}}\right) - \frac{\Delta P}{q_o}\left(\frac{A_{max} - A_c}{A_{max}}\right)$$

$$= \frac{W_a\,S_a\,(C_{f_e}\,\phi M_e - 1.0) - P_o\,(A_e - A_c)}{q_o\,A_{max}} + \frac{\Delta P}{q_o}\left(\frac{A_c - A_e}{A_{max}}\right)$$

$$= \frac{P_{t_o}\left(\frac{P_{t_c}}{P_{t_o}}\right) A_c\,(P\mathring{m}/P_t)_c\,S_a\,(C_{f_e}\,M_e - 1.0)}{\sqrt{T_{t_c}}\,q_o\,A_{max}} - \left(\frac{P_t}{q}\right)_o\left(\frac{p}{P_t}\right)_o\left(\frac{A_c}{A_{max}}\right)\left(\frac{A_e}{A_c} - 1\right)$$

$$+ \frac{\Delta P}{P_{t_o}}\left(\frac{P_t}{q}\right)_o\left(\frac{A_c}{A_{max}}\right)\left(1 - \frac{A_e}{A_c}\right)$$

Using $S_a = 2.42\,\sqrt{T_{t_c}}\,(1 + f/a)$ which is a good approximation,

$$\Delta C_t = \frac{A_c/A_{max}}{(q/P_t)_o}\left\{\frac{P_{t_c}}{P_{t_o}}\,(P\mathring{m}/P_t)_c\,2.42\,(1 + f/a)(C_{f_e}\,\phi M_e - 1.0) - \left(\frac{p}{P_t}\right)_o\left(\frac{A_e}{A_c} - 1\right) + \left(\frac{\Delta P}{P_{t_o}}\right)\left(1 - \frac{A_e}{A_c}\right)\right\} \qquad (13.1\text{-}10)$$

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

This equation was worked out for M_o = 1.5, 2.0, 2.5, and 3.0, again assuming a normal shock inlet and internal losses to the exit nozzle throat as in Figs. 13.1-1 and 13.1-2. The variation of the assumed boattail pressure coefficient is shown in Fig. 13.1-3. A_o/A_{max} is again assumed to be variable so as to place the shock on the rim.

Since at these high Mach numbers expansion to ambient P_o or base pressure very quickly leads to an $A_e > A_{max}$ when $A_b = A_{max}$, it was assumed that A_{max} equaled A_b in expansion. For this illustration, the complication of changing the basis for C_t is avoided.

The results shown in Fig. 13.1-3 indicate the rise in thrust coefficient as Mach number increases both for arbitrarily selected values of C_{f_e} = 1.00 and 0.97. No attempt is made to include the variation of C_{f_e} at off-design pressure ratios which is discussed in a later section. The values for A_c/A_b = 0.4 at M_o = 1.50 are slightly low because the pressures at that point indicate separation which normally raises C_{f_e} above unity.

Aside from the rise in the thrust as Mach number increases, a flattening of the C_t curve at high Mach numbers is quite apparent. This effect results from the fact that ϕM becomes asymptotic to 1.429 for γ = 1.40 or 1.591 for γ = 9/7 as M approaches infinity. For the fixed values of A_{max}/A_c, the fraction of possible ϕM decreases as M_o increases, which would also tend to make a curve for constant A_e/A_c flatter than one which expanded to ambient pressure. The flatter curves for A_c/A_b = 0.8 show this effect of reduced expansion.

The effect of a thrust loss in the divergent section of the nozzle represented by C_{f_e} is shown in Fig. 13.1-3. The tendency for this loss to be constant at various Mach numbers for a particular A_c/A_b arises from the relationship between P_{t_c}/P_{t_o} and q/P_{t_o}. The following equation for the loss of C_t caused by a reduction of C_{f_e} indicates this trend.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

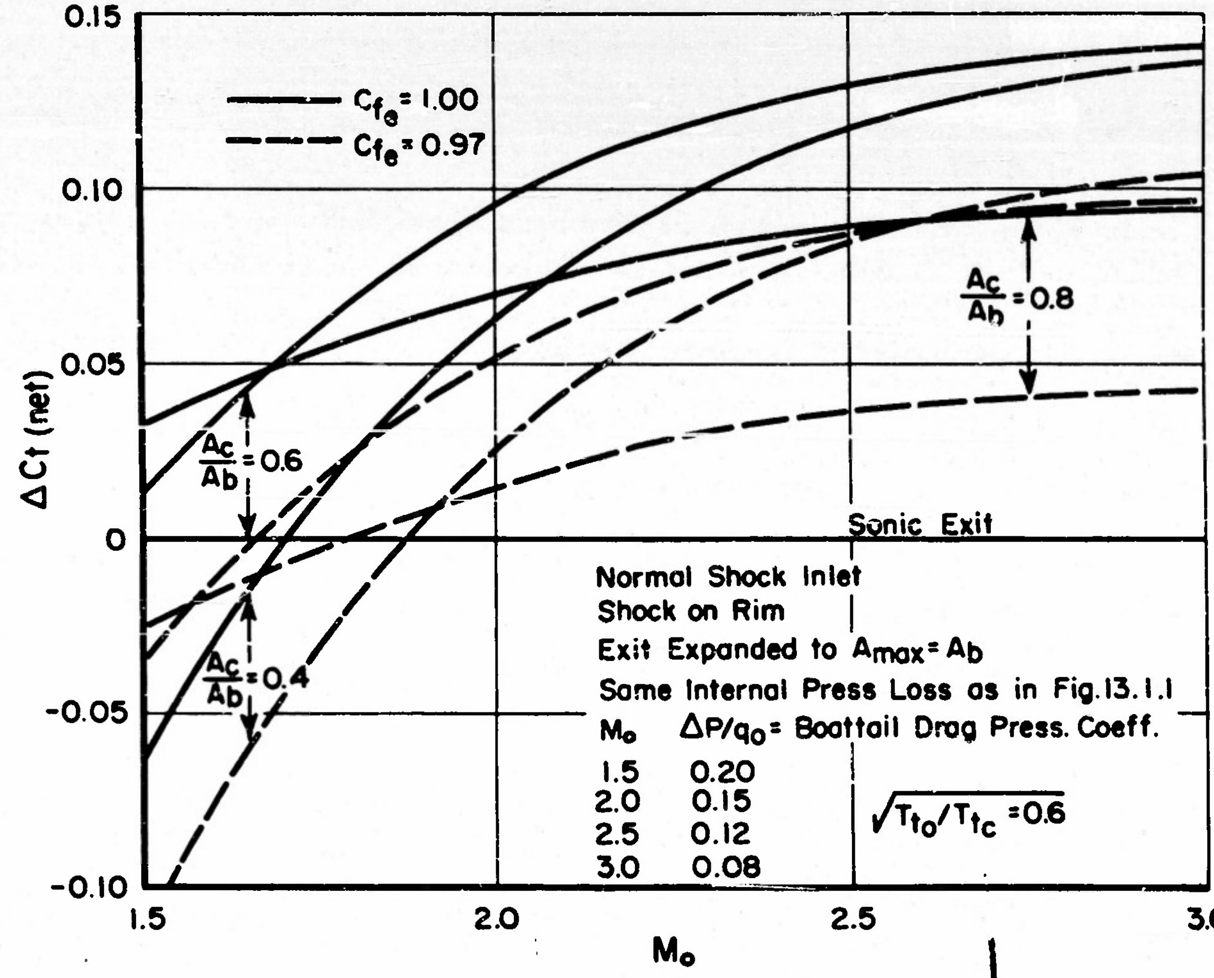


Fig. 13.1-3 EXAMPLE OF THRUST COEFFICIENT GAIN OF DIVERGENT EXIT OVER A SONIC EXIT--EXPANSION TO $A_{max} = A_b$

CONFIDENTIAL

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

$$C_t \text{ loss} = \frac{W_a \; S_a \; \phi M_e \; (1 - C_{f_e})}{q_o \; A_{max}}$$

$$= \frac{A_c/A_{max}}{(q/P_t)_o}\left(\frac{P_{t_c}}{P_{t_o}}\right) 2.42 \; (1 + f/a)(\phi M_e)(1 - C_{f_e}) \tag{13.1-11}$$

For the selected normal shock diffuser, the ratio $(P_{t_c}/P_{t_o})/(q/P_t)_o$ decreased slightly as Mach number increased. This effect shows up distinctly by comparison of the solid and dotted curves for $A_c/A_b = 0.8$. If a better diffuser had been selected (still using the low-drag burner) the ratio would have increased by about 50 per cent from $M_o = 1.5$ to 3.0. This would cause the loss of C_t from nozzle thrust loss to increase with flight Mach number

The values in Fig. 13.1-3 are shown for $\sqrt{T_{t_o}/T_{t_c}} = 0.6$ and would not change appreciably for other values of the temperature ratio. As noted in the discussion of Fig. 13.1-2 the percentage change of C_t would vary with the temperature ratio.

The Effect of Exit Constriction on Combustor Total Pressure Recovery

There is a total pressure loss in the combustor which arises from the burning as well as the drag of the combustor elements (see Chapter 2). The exit constriction also affects this total pressure loss which increases as: (a) total-temperature ratio across the combustor increases, (b) as combustor drag increases, or (c) as exit constriction ratio, A_c/A_b, increases.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

For a given combustor-drag coefficient and total temperature ratio the total pressure loss is roughly proportional to A_c/A_b as shown in Fig. 13.1-4

In addition to pressure recovery effects, the exit constriction or nozzle length can also affect the completeness of combustion. Considering the combustion to be a function of reaction rate and time, the exit-constriction ratio sets the time required for the gases to move a given distance from the flameholder to the exit. Where the reaction rate is sufficiently high for the combustion to be essentially complete before reaching the exit nozzle, the exit constriction ratio would, of course, have little or no effect on combustion efficiency. At lower reaction rates, where the combustion is only partly completed at the exit nozzle station, a moderate change in constriction ratio or nozzle length can make significant changes in the combustion efficiency at the exit plane.

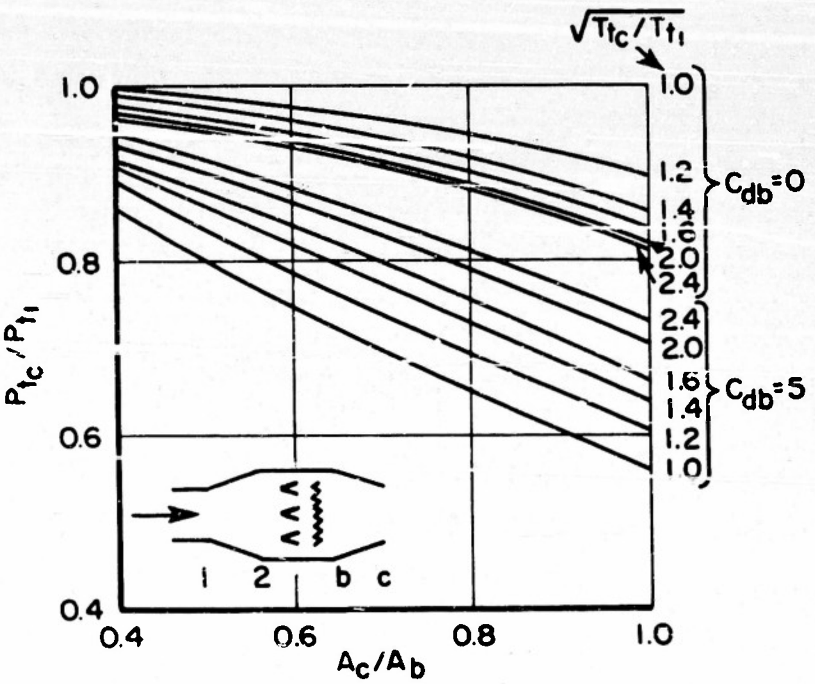
Summary of Exit Nozzle Effects

The following statements may be made regarding the relation of the exit nozzle to the complete ramjet powerplant.

1. Variations of the exit constriction are necessary for powerplant optimization.
2. Such optimization must consider the missile as a whole, particularly boattail drag.
3. Total pressure losses in the exit nozzle lower the C_t and I_f of a powerplant (except for swallowed shock).
4. Although C_t can be maintained by increasing the throat area to compensate for P_t losses, the I_f will be lowered.
5. Performance gains in C_t and I_f can be realized by using a divergent section downstream of the exit nozzle throat.

CONFIDENTIAL

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW



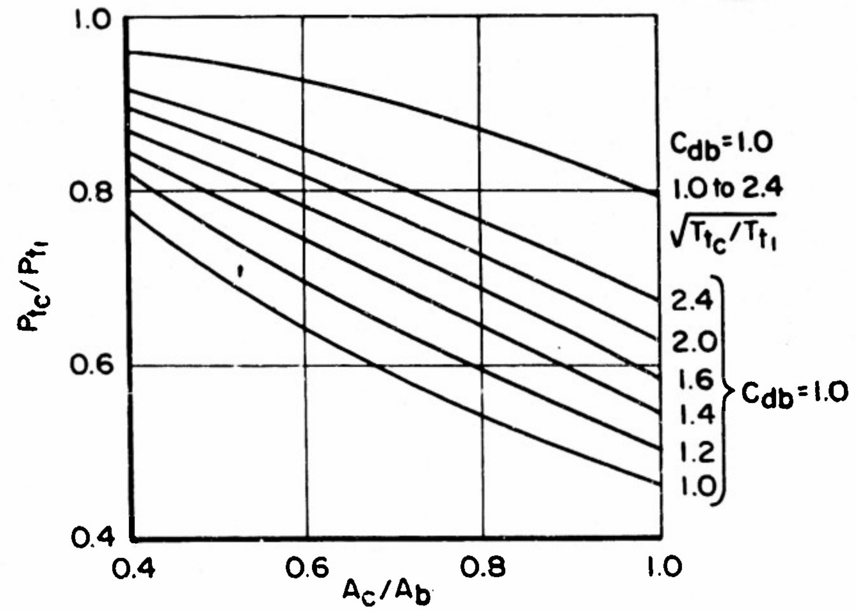


Fig. 13.1-4 EFFECT OF EXIT CONSTRICTION ON COMBUSTOR TOTAL PRESSURE RECOVERY--SONIC THROAT

The following conditions were $\gamma_1 = \gamma_2 = 1.40$, $\gamma_b = \gamma_c = 9/7$ with adiabatic flow.

CONFIDENTIAL

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

6. The gains, obtained with exit divergence, increase with Mach number, but the rate of increase of the gain decreases as Mach number increases.
7. Thrust losses in the divergent section decrease the C_t and I_f, an amount depending mainly on the constriction and expansion ratios and the relation of powerplant total pressure loss with flight Mach number.
8. Exit constriction increases the combustor total pressure recovery for a given combustor drag coefficient.
9. Seemingly small losses in the exit nozzle can cause large percentage changes in I_f and C_t for low values of C_t.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

13.2 PERFORMANCE OF CONVERGENT EXIT NOZZLES

Since both convergent and convergent-divergent exit nozzles are used with ramjets it is convenient to discuss these two types separately. A discussion of the sonic convergent section is applicable to the converging section of a convergent-divergent exit nozzle.

The main effort here is to describe the real flow within convergent nozzles in order to understand the nature and magnitude of the factors affecting nozzle discharge and stream thrust.

One-Dimensional Relations

The one-dimensional equations for a convergent channel indicate that the velocity increases as the area decreases and as the pressure ratio, P_{t_b}/P_c, increases up to the point where sonic velocity is attained at the minimum area or throat ($M_c = 1.0$). Further increase of the upstream total pressure only raises the general pressure level without affecting the throat velocity. The pressure ratio at which the sonic velocity is obtained is commonly referred to as the critical pressure ratio which can be defined in terms of the ratio of specific heats, γ.

$$(P_t/P)_{cr} = \left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma}{\gamma-1}} \qquad (13.2\text{-}1)$$

For almost all ramjet applications the exit-nozzle pressure ratio is sufficient to produce sonic velocity at the throat. For this reason, and since the conditions at the exit are of interest in missile-performance calculations, it is most convenient

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

to express the flow and thrust for convergent nozzles in terms of conditions at the throat denoted by subscript "c".

$$W = \rho_c A_c V_c = \frac{P_c A_c \overset{\circ}{m}_c}{\sqrt{T_{t_c}}}$$

$$= \frac{P_{t_c} A_c (P\overset{\circ}{m}/P_t)_c}{\sqrt{T_{t_c}}} \qquad (13.2\text{-}2)$$

$$F_c = \overset{\circ}{m}_c V_c + P_c A_c = P_c A_c (f/P)_c$$

$$= P_{t_c} A_c (f/P_t)_c$$

$$= W_a (S_a)_c \phi M_c \qquad (13.2\text{-}3)$$

The various forms of these equations are identical for one-dimensional isentropic flow. Where pressure and velocity profiles exist (as in real nozzles) the equations are not identical, the differences being dependent on the particular parameter integrated over the profile. The one-dimensional isentropic values of the various Mach number functions for the above equations are tabulated below for sonic throat velocity ($M_c = 1.00$) for two values of specific heat ratio commonly used for cold and hot gas streams.

TABLE 13.2-1

	$\gamma = 1.40$ (cold)	$\gamma = 9/7$ (hot)
$\overset{\circ}{m}$	1.0066	0.9414
$P\overset{\circ}{m}/P_t$	0.5318	0.5162
f/P	2.400	2.286
f/P_t	1.268	1.253
ϕM	1.000	1.000

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

In the one-dimensional equations it is assumed that the stream properties of pressure, velocity, Mach number, etc., do not vary in a plane normal to the mean flow direction. This assumption is not entirely applicable to the flow in exit nozzles. The practical convenience of the one-dimensional equations demands their use, however. The description of real flows is accomplished by the use of coefficients applied to the one-dimensional isentropic values which will be referred to as "ideal."

Deviations of Real Flows

Precise definition of the deviation of the discharge and thrust of real flows from ideal values would require that the exact nature of the pressure and velocity variations throughout the nozzle be known. However, since these variations are not generally known, the most practical course is to investigate, to the extent allowed by present theory and experiment, the basic causes of the deviation of real flows from the ideal. The purpose of such investigations would be to determine, insofar as possible, the magnitude of the deviations.

The assumption of frictionless flow is violated at the solid boundary of the nozzle where a boundary layer exists. The friction in this boundary layer causes a loss of both discharge and thrust. Although there will be some frictional effects in the core of the flow outside the boundary layer region, these effects should be negligible for most ramjet exit nozzles as compared to the pressure and inertia forces. The negative axial pressure gradient along the wall helps prevent separation of the boundary layer.

There may also be variation of the flow properties normal to the mean flow direction. Since the flow near the wall ahead of a convergence must be displaced radially inward in order to

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

pass through the constriction, a force is required to produce the radial acceleration. This implies that at the beginning of a constriction, there must be a pressure gradient normal to the flow direction such that the pressure increases as the distance from the nozzle axis increases. Since it would be physically impossible for the flow to continue in a radially inward direction to a point, there must be a region where parallel flow to the axis is resumed. Turning the flow parallel to the axis requires a radially outward force or a static pressure gradient such as to decrease the pressure at increasing distances from the nozzle axis. These variations of stream properties normal to the mean flow direction are a fundamental and necessary deviation from one-dimensional flow in exit nozzles.

In some cases, the minimum flow area may be less than the minimum geometrical area because of the radially inward momentum of the stream at the wall of the minimum section. Since there is no way at present to determine the true minimum flow area, the only practical course is to base the flow on the known geometrical area.

There may be additional heat release from combustion in the exit nozzle which, in most cases, is not evaluated separately from the burning in the combustion chamber. Recognition of the effects of burning materially aids in the interpretation of physical observations.

Thus there are four important deviations of a real flow from the "ideal" one-dimensional flow assumed for convenience.

1. Wall friction,
2. Variations of flow variables across a section,
3. Jet contraction beyond the minimum geometrical area, and
4. Burning in the nozzle.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

Definition of Coefficients

There are two characteristics of the flow through an exit nozzle which are of principal importance, the quantity of flow under given conditions and the exit stream thrust produced by this flow. In the case of a convergent exit nozzle, the throat stream thrust is of interest for flight performance calculations. For a convergent-divergent nozzle both the throat and exit stream thrust are of interest.

The discharge and throat stream thrust for an isentropic one-dimensional, or "ideal" flow in a convergent nozzle are given by Eqs. (13.2-2) and (13.2-3), respectively. The exit stream thrust of a convergent-divergent nozzle is described in a later section.

The discharge coefficient is defined by the ratio of actual to ideal flow. Since the ideal flow is assumed to be isentropic it makes no difference in the calculation of the ideal flow whether upstream or throat total pressures and temperatures are used. With a real ramjet exit nozzle, however, the upstream and throat values of total pressure and temperature may be different. Total pressure losses and total temperature losses or gains are a result of the exit nozzle and should be charged to the nozzle so that upstream values should be used for definition of the discharge coefficient.

Using the subscript "b" for conditions ahead of the nozzle, "c" for the throat and "i" for ideal, the following equation defines the discharge coefficient,

$$C_d = \frac{W}{W_i} = \frac{W \sqrt{T_{t_b}}}{P_{t_b} A_c (P\mathring{m}/P_t)_{c_i}} \qquad (13.2\text{-}4)$$

where W is the actual flow. Values of $(P\mathring{m}/P_t)_{c_i}$ for a sonic

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

velocity at the throat can be obtained from Table 13.2-1. For less than sonic velocity the value of $(\dot{P}m/P_t)_{c_i}$ can be calculated for the throat Mach number as determined by the nozzle pressure ratio.

Since the ramjet designer is more apt to know and be concerned with total pressures and temperatures than with the static values, C_d is expressed in terms of total values. Thus no "velocity of approach" factor is used in Eq. (13.2-4).

There are several expressions for the thrust or jet reaction of a stream of fluid. The form of these expressions varies with the particular application. For rocket analysis, a thrust equation is commonly used in which a velocity coefficient is applied to the mV term and the pressure terms are assumed to be equal to isentropic one-dimensional values. However, for ramjet analysis it is generally more convenient to use the stream thrust since the net axial force on the fluid flowing between two stations is the difference in the respective stream thrusts at the two stations. Since a part of the throat stream thrust deviation is caused by the reduction of flow already expressed by C_d, the throat stream thrust coefficient need express only the remainder of the stream thrust deviation. Using the actual airflow,

$$C_{f_c} = \frac{F_c}{W_a \, (S_a)_c \, \phi M_{c_i}} = \frac{F_c}{C_d \, (F_{c_i})}$$

$$= \frac{\phi M_c}{\phi M_{c_i}} \qquad (13.2\text{-}5)$$

$(S_a)_c$ is the actual S_a at the throat. The reason for using the throat S_a is that most ramjet tests evaluate combustor performance at the throat of a sonic exit nozzle. As discussed in a later section, this definition renders C_{f_c} independent of burning in the subsonic section of the nozzle.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

The Effects of Wall Friction

One of the deviations of a real flow from an ideal flow is friction in the boundary layer along the nozzle wall. Fortunately, for most ramjet applications, these frictional effects do not give rise to large deviations of discharge or thrust from "ideal" values. However, for high efficiency ramjet combustors even seemingly small losses become important. An attempt is made in this section therefore to evaluate these relatively small frictional effects.

Most of the work on nozzles has been concerned with the establishment of standardized nozzles for flow measurement purposes. Such nozzles have well rounded entry sections and a parallel throat section so that the main discrepancy from ideal flow is the effect of friction in the boundary layer. Their discharge coefficient becomes very nearly constant at a pressure ratio slightly above the critical pressure ratio. It is typical of well-contoured nozzles to have a very slight increase in discharge coefficient as pressure ratio is increased (constant discharge pressure). It is probable that the slight increase is caused by the increased Reynolds number which causes a smaller boundary layer thickness.

A method has been developed [2] for predicting the discharge coefficient for well-rounded nozzles. This method is based upon the apparent laminar friction coefficients in the transition length of a straight tube. The discharge coefficient is calculated from the friction factor. A length to throat diameter ratio, (x/D_c), is described below.

$$C_d = \frac{1}{\sqrt{1 + 4f_{app.}(x/D_c)}} \qquad (13.2\text{-}6)$$

CONFIDENTIAL

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

Values of $4f_{app.}$ (x/D_c), Ref. [2], are given in Fig. 13.2-1 for various values of $(x/D_c)/R_c$ where R_c is the throat Reynolds number based on diameter. The value of (x/D_c) is composed of the length to throat diameter ratio of the cylindrical throat section, if one exists, plus an effective $\Delta(x/D_c)$ for the bellmouth. Letting D be the diameter of any section of the bellmouth:

$$\Delta(x/D_c) = \int \left(\frac{D_c}{D}\right)^5 d\left(\frac{x}{D_c}\right). \tag{13.2-7}$$

For a circular arc-wall profile with radius equal to D_c the $\Delta(x/D_c)$ is about 0.4. For a circular arc profile with radius equal to 0.5 D_c the $\Delta(x/D_c)$ is about 0.25. For an ellipse with semi-axes D_c axial and 0.7 D_c radial the $\Delta(x/D_c)$ is about 0.6.

Fairly good agreement is obtained over a very wide range of Reynolds numbers ($R_c \approx 1$ to 10^6) when the predicted values are compared with experimental discharge coefficients of ASME flow nozzles. Although the presence of a laminar boundary layer may be questionable in a ramjet exit nozzle, the turbulent friction should be higher. The above values would therefore appear to represent maximum values of the discharge coefficient. For a throat Reynolds number of 10^6 and $(x/D_c) = 1.0$ the maximum value of C_d would be about 0.99.

The deviation of throat stream thrust from the ideal value can be estimated from the apparent friction factor. Using the apparent friction factor from Fig. 13.2-1 to obtain the throat stream thrust coefficient,

$$\begin{aligned} C_{f_c} &= \frac{F_c}{C_d(W_{a_i} \; S_a \; \phi M_{c_i})} \\ &= \frac{F_{c_i} - \Delta PA_c}{C_d(F_{c_i})} = \frac{1}{C_d}\left(1 - \frac{\Delta PA_c}{F_{c_i}}\right) \\ &= \frac{1}{C_d}\left(1 - \frac{4f_{app.}(x/D_c)\, q_c\, A_c}{(P_{t_c}\, A_c\, f/P_{t_c})_i}\right) \end{aligned} \tag{13.2-8}$$

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

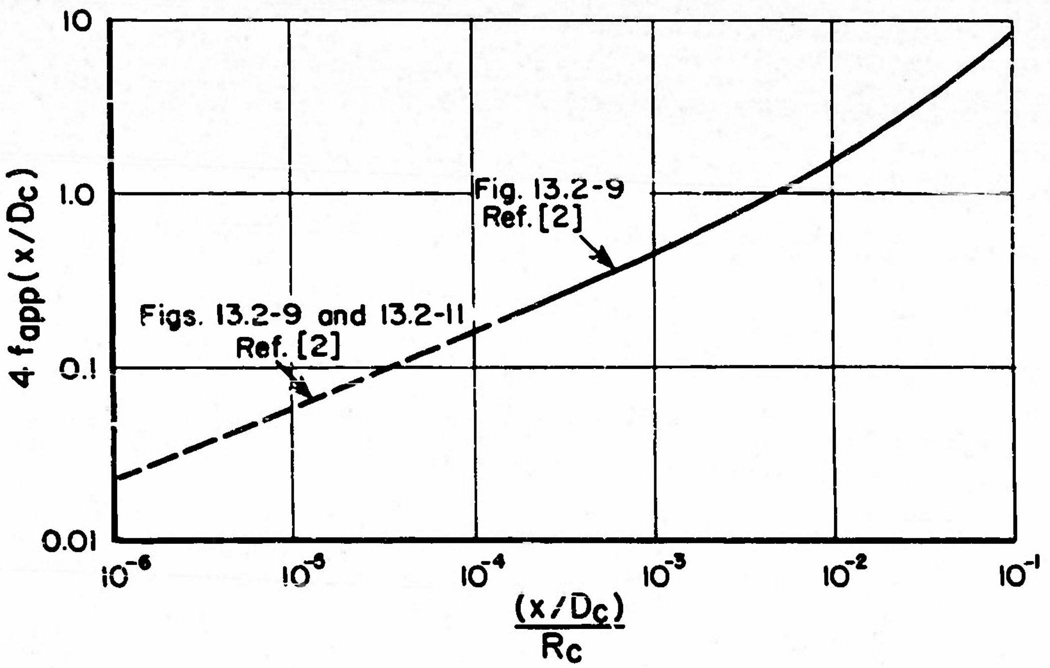


Fig. 13.2-1 APPARENT FRICTION FACTOR FOR LAMINAR INLET REGION

CONFIDENTIAL

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

For sonic values at the throat ($\gamma = 9/7$), $R_c = 10^6$ and $x/D_c = 1.0$ C_d from Eq. (13.2-6) = 0.99 ,

$$C_{f_c} = 1.01 \left(1 - \frac{0.022\ (0.352)}{1.253}\right) = 1.01\ (1 - 0.006)$$

$$= 1.004.$$

Since the flow is reduced by one per cent the actual throat stream thrust is reduced by the factor (0.99)(1.004) = 0.994 from the "ideal" value. The exit stream thrust coefficient exceeds unity slightly because ϕM in the boundary layer exceeds unity, the minimum value of ϕM being 1.0 at $M = 1.0$.

The friction factor for turbulent flow in pipes at the assumed Reynolds number is about the same as the value of $4f_{app.}$ used above. Although the friction factor at the nozzle inlet is expected to be slightly higher than for developed-pipe flow, a Reynolds-number range is approached in ramjet applications where a reasonable surface roughness determines the friction factor rather than Reynolds number.

Another method of evaluating the boundary-layer effect is by calculation of the displacement thickness and momentum thickness of the boundary layer. Since the nozzle inlet presents numerous complications, one must resort to flat-plate equations as an approximation.

A rough idea of the magnitude of the error involved by the flat plate approximation can be obtained by considering the complicating factors individually. According to Mangler [3], the displacement thickness on the external surface of a cone at supersonic speeds is $1/\sqrt{3}$ as great as on a flat plate (friction is $\sqrt{3}$ times the flat plate value). Since the boundary-layer flow covers less area as it moves downstream instead of more area as in the case of the cone, the effect should be opposite to that of a cone so that contraction should increase the displacement thickness slightly. Weil, [4], indicates that a

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

favorable (decreasing) pressure gradient decreases the boundary layer thickness and increases the skin friction. Van Driest [5], indicates that transfer of heat from the exhaust gas stream will decrease the displacement thickness and will increase the friction.

Wall curvature effects should be negligible so long as the wall radius of curvature is large compared to the boundary layer thickness. Thus there are several complicating effects which, to a degree, are compensating. Fortunately, since the corrections to thrust and discharge are small compared to unity, great accuracy is not required.

The displacement thickness, δ^*, (which is the thickness of a stagnant region which has the same velocity defect as the integrated velocity defect over the entire boundary layer) is used to evaluate the discharge coefficient. The momentum thickness, θ, which similarly measures the momentum defect is used to determine the loss of throat-stream thrust. Setting the discharge coefficient equal to the ratio of the net area to the full area, and neglecting a second order term, the discharge coefficient can be expressed by

$$C_d = \frac{(D_c - 2\delta^*)^2}{(D_c)^2} = \frac{D_c^2 - 4\delta^* D_c + \ldots}{D_c^2}$$

$$= 1 - 4\delta^*/D_c, \tag{13.2-9}$$

where δ^* is the boundary layer displacement thickness.

Using the formula for boundary layer growth for turbulent flow on a flat plate as an approximation,

$$\delta^* = 0.125\,\delta = \frac{0.041\,L}{(R_L)^{0.2}}, \tag{13.2-10}$$

where R_L is based on the length, L, from the leading edge of a flat plate. Using this length as the nozzle inlet length

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

since concavity at the beginning of the constriction should insure transition to turbulent flow

$$\delta^* = \frac{0.041\ L}{(R_c)^{0.2}\ (L/D_c)^{0.2}}\ ,\ \text{or}$$

$$\frac{\delta^*}{D_c} = \frac{0.041\ (L/D_c)^{0.8}}{(R_c)^{0.2}} \qquad (13.2\text{-}11)$$

Substitution in Eq. (13.2-9) gives,

$$C_d = 1 - \frac{0.16\ (L/D_c)^{0.8}}{(R_c)^{0.2}} \qquad (13.2\text{-}12)$$

Again for R_c equal to 10^6 and $L/D_c = 1.0$ the value of C_d is about 0.99.

For turbulent flow the ratio of momentum thickness, θ, to the displacement thickness δ^* is about 0.80.

$$\frac{\theta}{D_c} = \frac{0.033\ (L/D_c)^{0.8}}{(R_c)^{0.2}} \qquad (13.2\text{-}13)$$

Using the reduced area $(\pi/4)(D_c - 2\theta)^2$ to obtain the actual exit stream thrust,

$$C_{f_c} = \frac{F_c}{C_d(F_{c_i})} = \frac{1}{C_d}\left(\frac{D_c - 2\theta}{D_c}\right)^2$$

$$= \frac{1}{C_d}\ (1 - 4\theta/D_c + \ldots). \qquad (13.2\text{-}14)$$

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

For $R_c = 10^6$ and $L/D_c = 1.0$, and $C_d = 0.99$ from Eq. (13.2-12)

$$C_{f_c} = 1.01\ (1 - 0.007) = 1.003.$$

Thus the actual throat stream thrust is 0.99(1.003) or 0.993 of the "ideal" value.

While no claim is made for great accuracy of these calculations, they at least indicate the small magnitude of boundary-layer effects on the discharge and exit-stream thrust of converging exit nozzles for the relatively high Reynolds numbers associated with ramjets.

For large constriction ratios, the errors in thrust and discharge may become relatively large, particularly where there is no definite break in the wall slope at the nozzle entrance. This arises from the consideration that a boundary layer will have developed upstream of the nozzle which will affect the boundary layer in the nozzle. This type of nozzle will probably require tests with simulation of the upstream boundary layer if accurate results are desired.

There are very few data with which to check the above calculations. Comparison of two nozzles [9] (see nozzles A and B of Fig. 13.2-2) having well-rounded entries and parallel throat sections indicated that the longer nozzle ($L/D_c = 1.0$) had a lower discharge coefficient than the shorter nozzle ($L/D_c = 0.5$). The fact that the longer nozzle had an average discharge coefficient of 0.990 and the shorter one 0.995 indicates evidence of increased friction in the longer nozzle since it was almost exactly twice as long as the shorter nozzle. The average experimental values of C_{f_c} were 1.001 for the longer nozzle and 0.998 for the shorter nozzle. The above equations would give 1.003 and 1.001 respectively. The value of 0.998 is

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

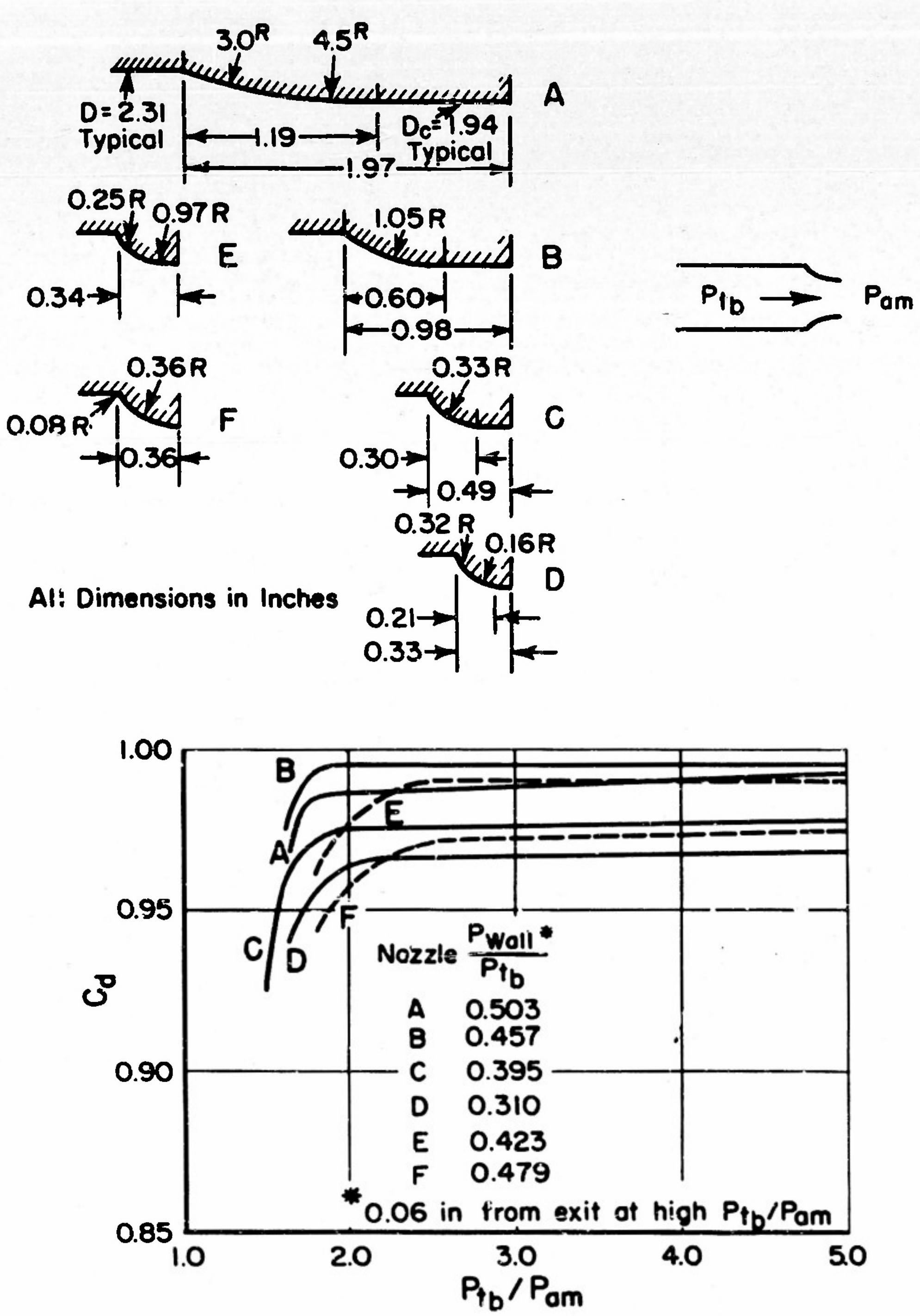


Fig. 13.2-2 DISCHARGE COEFFICIENTS FOR ROUND APPROACH EXIT NOZZLE

The data were obtained from Ref.[9].

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

clearly experimental error since ϕM has a minimum value of 1.0 at $M = 1.0$. If the error were systematic, the difference in C_{f_c} between the two nozzles would be more reliable than the absolute values. The theoretical and experimental differences agree well within the experimental accuracy. The Reynolds numbers in these cold flow nozzle tests approximated that of a 24-inch-diameter combustor with burning.

It should be noted that the entire stream cannot be strictly treated as a one-dimensional stream with a total pressure loss to account for friction, although, for most cases, the error arising from such treatment will not be large. For example, assuming the total temperature and the $\dot{P}m/P_t$ ratio to be equal to the ideal values, the previous value of C_d for 0.99 [Eq. (13.2-12)] could be interpreted as a one per cent total pressure loss over the entire area. If f/P_t were also assumed equal to the ideal value, this same one per cent total pressure loss would cause a one per cent loss of stream thrust. The above equations and experimental results indicate the stream thrust loss is only 0.6 per cent. For most ramjet calculations this difference is negligible so that a total pressure loss interpretation would be acceptable. For high efficiency combustion systems, however, a more detailed analysis may be warranted since even small losses can markedly affect the over-all performance.

Friction effects in a convergent nozzle can be summarized as follows:

1. Friction causes a reduction of discharge and stream thrust.
2. The stream thrust loss is not as great as the discharge loss.
3. These losses for most ramjet exit nozzles will be less than one per cent for nozzle inlets which are less than one throat diameter long ($R_c \approx 10^6$).

CONFIDENTIAL

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

4. Although these losses may be approximated roughly by means of a total pressure loss, exact solution will require detailed study of the boundary layer.

The Effects of Wall Curvature at the Throat

There are several effects resulting from the shape of the wall contour which are important in the determination of convergent nozzle performance. Since length limitations are usually quite severe the ramjet exit nozzle is usually made much shorter than nozzles used for other purposes.

The nozzles considered up to now have had well-rounded inlets and throat sections with little or no wall curvature; that is, a parallel throat section. With these comparatively long nozzles, changes in the flow direction within the nozzle are rather gentle with the result that pressure, velocity, and Mach number profiles normal to the nozzle axis are relatively flat. The necessarily more rapid changes in flow direction obtained in short nozzles result in pressure, velocity, and Mach number profiles normal to the axis which are far from flat.

The shape and location of the sonic surface near the throat are discussed in Refs. [6,7, and 8]. The shape of the sonic velocity profile (a section through the surface where the Mach number is unity) was determined [7] to be parabolic to a first approximation. The ratio of the radius of curvature of the wall at the throat to the throat diameter was found to be the controlling parameter. Figure 13.2-3 shows the shape and location of the sonic profile for several degrees of wall curvature at the throat. The curve is parabolic and crosses the plane of minimum area at a distance equal to 0.707 of the throat radius. Since the equations from which these curves were obtained are linearized, the results for sharp wall curvatures will not be as accurate as for the gentler curvatures.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS. TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

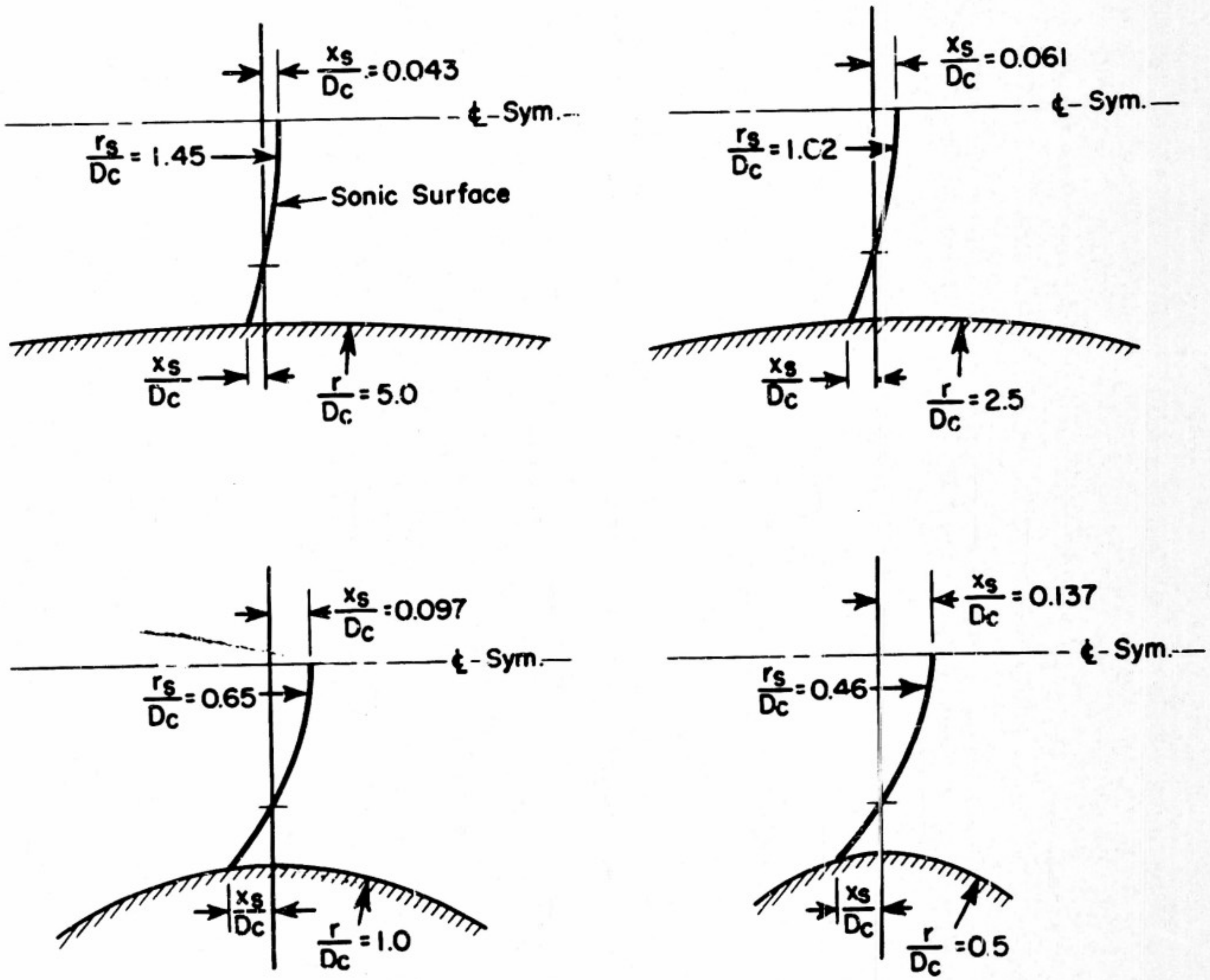


Fig. 13.2-3 **PROFILES OF THE SONIC SURFACE AS A FUNCTION OF WALL CURVATURE AT THE THROAT**

An axially symmetric nozzle was used. The data were obtained from Refs. [7] and [8].

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

Tests of a nozzle having a 70 per cent constriction and an $r/D_c = 0.18$ revealed that the pressure on the centerline reached the $M = 1.0$ value ($0.528\ P_t$) at a distance of $0.12\ D_c$ downstream of the minimum area. Apparently, for sharp-wall curvatures, the sonic surface profiles of Refs. [7] and [8] may be too strongly curved.

The wall and centerline Mach numbers at the minimum geometrical area are also calculated with linearized equations in Ref. [8]. Figure 13.2-4 shows the variation of these Mach numbers with radius of wall curvature for an axially symmetric nozzle.

A correction for discharge is given in Ref. [6], which is dependent upon the ratio of the radius of curvature of the wall at the throat, r, to the throat diameter, D_c. The equation results from a first approximation and is restricted to small wall slopes and radii of curvature of the order of D_c or greater.

$$C_d = 1.0 - \frac{\gamma + 1}{384\ (r/D_c)^2} \qquad (13.2\text{-}15)$$

$$= 1.0 - \frac{1}{160\ (r/D_c)^2} \quad \text{for } \gamma = 1.40$$

For an axially symmetrical nozzle having a hyperbolic wall profile with $r_c/D_c = 1.67$ at the throat, the Mach number at the wall is 1.08 and at the centerline is 0.92. The discharge coefficient would be 0.998. Comparison of theoretical and experimental Mach numbers for $r_c/D_c = 2.5$ gave relatively good agreement up to the throat. (The wall Mach number rose to about 1.2, however, at a distance of about 0.1 of the throat diameter downstream of the throat.)

A similar treatment in Ref. [7] yields substantially the same results. The shape of the sonic velocity profile is determined to be parabolic to a first approximation. The axial

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

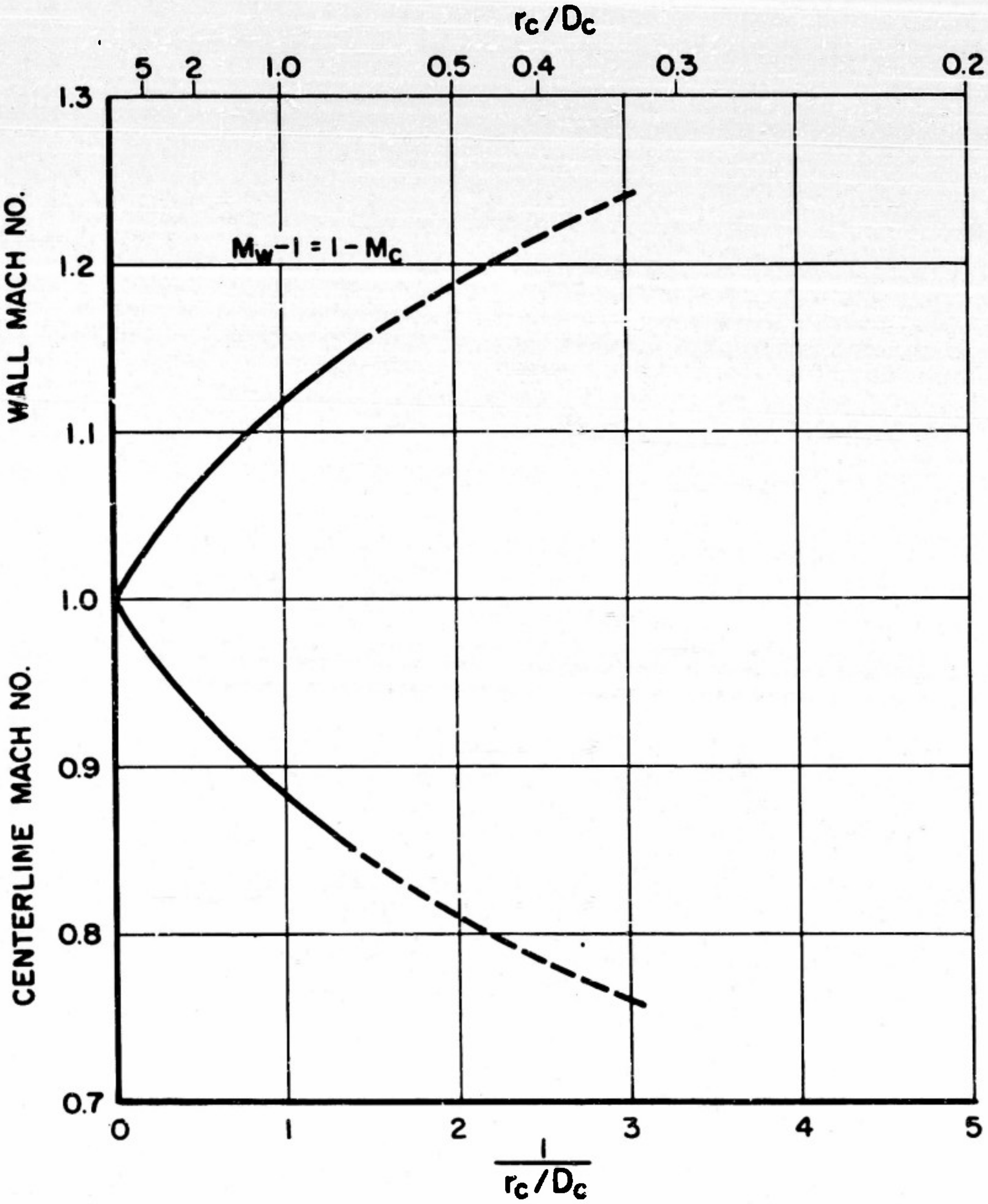


Fig. 13.2-4 EFFECT OF WALL CURVATURE AT THE THROAT ON THROAT MACH NUMBER PROFILE

An axially symmetric nozzle was used. The data were obtained from Ref. [8].

CONFIDENTIAL

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

velocity gradient of an axially symmetrical nozzle is shown to be about 40 per cent greater than that for a two-dimensional nozzle.

Reference [8] presents the wall and centerline Mach numbers at the throat plane of two-dimensional and axially symmetrical nozzles from which a discharge correction (to be applied to one-dimensional flow equations) is obtained. The discharge correction depends only on the ratio r/D_c at the throat, although discontinuities in the radius of curvature render the interpretation of the results uncertain. The following is an approximate formula which fits the tabulated values ($r/D_c > 0.75$) in Ref. [8] for an axially symmetrical nozzle and $\gamma = 1.40$.

$$C_d = 1.0 - \frac{1}{66.7\ (r/D_c)^{1.8}} \qquad (13.2\text{-}16)$$

The change in C_d with γ is shown to be very slight. At $r/D_c = 1.0$, Ref. [8] would give a C_d of about 0.985 as compared to 0.994 obtained from Ref. [6]. It is stated in Ref. [8] that, for an axially symmetric nozzle with gentle curvatures, the wall Mach number will exceed unity by the same amount the centerline Mach number is less than unity. Comparison with Ref. [6] at $r/D_c = 1.67$ gives exact agreement.

Figure 13.2-5 compares Eq. (13.2-15) from Ref. [6] and the reciprocal of the discharge correction from Ref. [8]. Also shown in Fig. 13.2-5 is the integration of $\mathring{P}m/P_t/(\mathring{P}m/P_t)_{M=1.0}$ over the throat area, assuming that the Mach number profile is parabolic and that the decrement below $M = 1.0$ at the centerline equals the increment above $M = 1.0$ at the wall.

$$C_d = \frac{1}{(\mathring{P}m/P_t)_{M=1.0}} \int_0^1 (\mathring{P}m/P_t)\ dA/A \qquad (13.2\text{-}17)$$

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

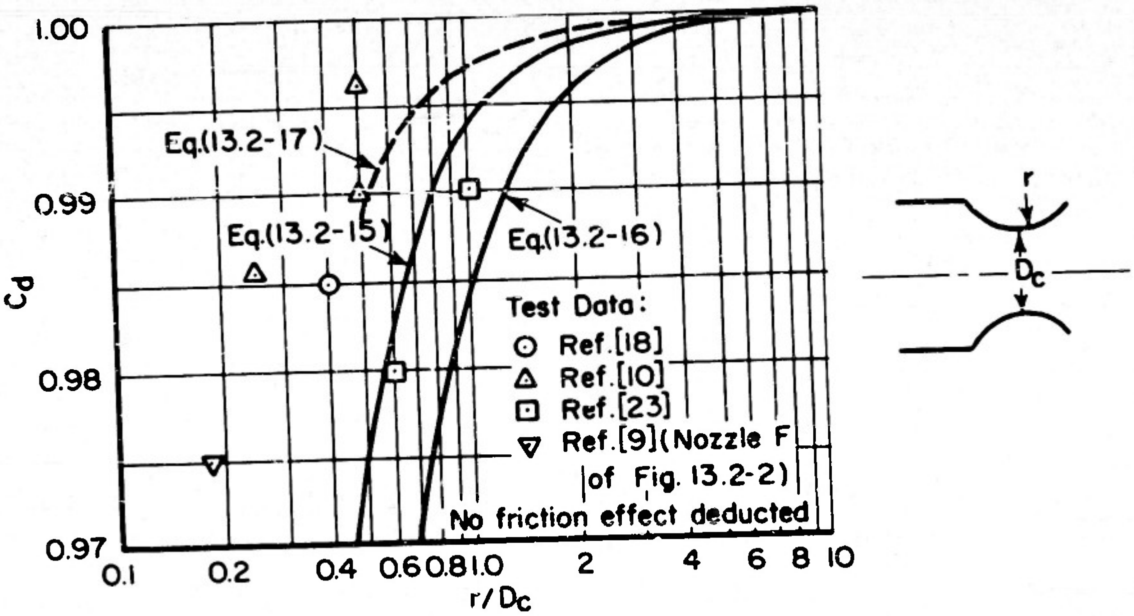


Fig. 13.2-5 EFFECT OF WALL CURVATURE AT THROAT ON DISCHARGE COEFFICIENT

CONFIDENTIAL

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

The values of the wall and centerline Mach numbers were taken from Fig. 13.2-4. Although Eqs. (13.2-16) and (13.2-17) should agree, it is apparent that they do not. The reason is unknown.

These effects occur in a completely isentropic flow and do not depend on viscous or total pressure losses.

Comparison of Fig. 13.2-5 with experimental results is difficult because of test inaccuracies and the fact that many nozzles do not have a constant radius of curvature of the wall near the throat. The comparisons shown in Fig. 13.2-5 are for nozzles which have a constant radius of curvature for a considerable distance upstream of the throat.

It is stated in Ref. [18] that a considerable deterioration of the discharge coefficient shows up only when the profile radius of curvature at the throat is less than 0.5 D_c and that the intake angle ahead of the throat has practically no influence on the losses in the nozzle intake. Data for nozzles having a 60-degree convergent total-angle cone followed by a throat-generating radius of about 0.4 D_c gave an average discharge coefficient of 0.985. The constriction ratio was about 33 per cent and the throat Reynolds number varied from about 6×10^5 to 3×10^6.

Tests carried out on nozzles having a conical constriction of 90-degree total angle followed by a throat-generating radius are reported in Ref. [10]. Changing the throat-generating radius from 0.5 D_c to 0.25 D_c caused only about 0.4 per cent decrease in the average discharge coefficient (0.990 to 0.986). Decreasing the intake total angle from 90 to 60 degrees with a throat-generating radius of 0.5 D_c increased the discharge coefficient 0.7 per cent from 0.990 to 0.997. These nozzles all had a 50 per cent constriction ratio and a throat Reynolds number of about 2×10^6 based on the throat diameter.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

Recent tests reported in Ref. [9] are given in Fig. 13.2-2. The longest nozzle did not have the highest discharge coefficient. The throat Reynolds number for these tests was 1×10^6 to 3×10^6 and the constriction ratio was 70 per cent. The two longest nozzles (A and B) were previously described in the section on the effects of wall friction. Since the shorter nozzles (C, D, and E of Fig. 13.2-2) have discontinuities in the wall curvature just upstream of the throat, they cannot be compared directly to Fig. 13.2-5.

Although it is shown in Ref. [8] that the wall profile effect on the discharge coefficient results only from the throat curvature with zero wall slope at the throat, it seems quite probable that the upstream curvature also exerts an influence on C_d, although the magnitude of the influence probably decreases with increasing distances upstream of the throat as the cross-sectional area increases.

While there is considerable scatter between the theoretical and test data it appears that the curves have the correct shape. When the r/D_c is less than 0.5 to 1.0 the discharge falls off rapidly and will probably be quite sensitive to small changes in the inlet-wall shape. The test values of C_d appear to agree as well with Eq. (13.2-17) as any other. It might be well to note that the test values from Ref. [23] used in Fig. 13.2-5 are probably somewhat low because no deduction was made for friction which is estimated to be from 0.5 to 1 per cent. Friction for the other test values is probably of the order of 0.2 per cent or less.

The stream-thrust loss can be evaluated in a manner similar to that of Eq. (13.2-17). Since the total pressure is constant, the stream thrust can be expressed in terms of f/P_t integrated over the flow area. An expression which appears to be more useful, however, is the ratio of the percentage loss of stream thrust to the percentage loss of flow.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

Assuming that unidimensional equations can be applied to a small portion of the flow and that this flow is at an arbitrary Mach number instead of M = 1.0 at the minimum geometrical area, the percentage airflow loss is

$$\frac{W_{a_{loss}}}{W_{a_i}} = 1 - \frac{W_a}{W_{a_i}} = \frac{(\overset{\circ}{P}m/P_t)_i - (\overset{\circ}{P}m/P_t)}{(\overset{\circ}{P}m/P_t)_i}$$

$$= 1 - C_d, \qquad (13.2\text{-}18)$$

where i in this case is the value at M = 1.0. Similarly the stream thrust loss is

$$\frac{F_{c_{loss}}}{F_{c_i}} = 1 - \frac{F_c}{F_{c_i}} = \frac{(f/P_t)_i - (f/P_t)}{(f/P_t)_i}$$

$$= 1 - C_d\, C_{f_c}. \qquad (13.2\text{-}19)$$

Denoting the ratio of the stream thrust loss to the airflow loss by N results in the following equation:

$$N = \frac{1 - C_d\, C_{f_c}}{1 - C_d} \qquad (13.2\text{-}20)$$

It is shown in Fig. 13.2-6 how N varies for Mach numbers near unity. Except for a very small region of M = 0.98 to 1.02, the percentage loss of stream thrust is about 60 per cent of the percentage flow loss.

If one were to obtain an integrated value of N, the throat stream thrust coefficient could be obtained from C_d.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

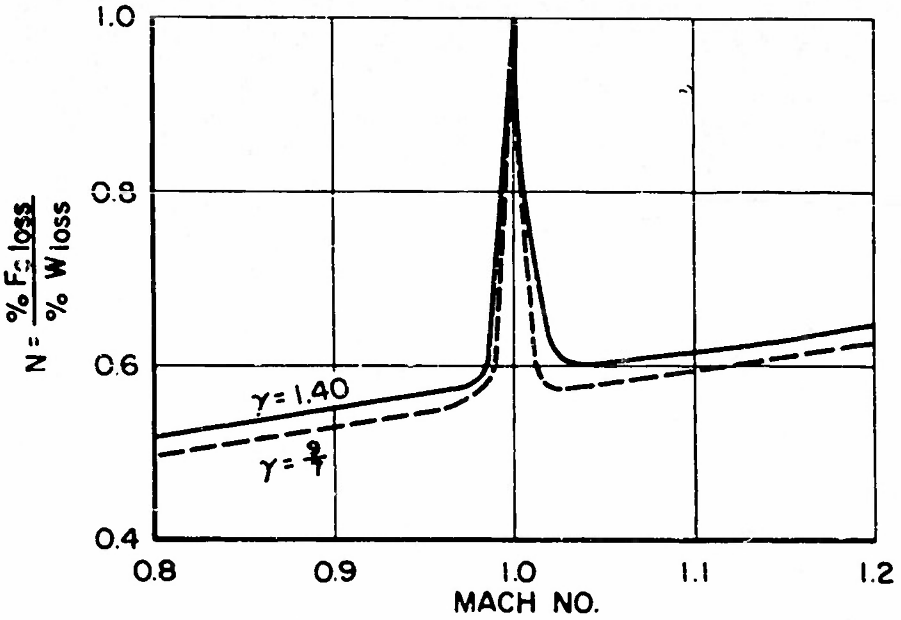


Fig. 13.2-6 RATIO OF PERCENTAGE STREAM THRUST LOSS TO PRESSURE FLOW LOSS vs MACH NUMBER

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

$$C_{f_c} = \frac{1 - N(1 - C_d)}{C_d} \tag{13.2-21}$$

Letting ΔM denote the deviation of the Mach number from $M = 1.0$, integrated values of N are shown in Fig. 13.2-7. The curves were first calculated as a direct integration of Fig. 13.2-6 from $-\Delta M$ to $+\Delta M$. This was equivalent to assuming a linear Mach number distribution from $1 - \Delta M$ to $1 + \Delta M$ in a two-dimensional duct. A series of integrations was also carried out for an axially symmetric duct assuming a parabolic Mach number profile from $1 - \Delta M$ at the centerline to $1 + \Delta M$ at the wall (see Fig. 13.2-4). The curves for the two types of flow were essentially identical.

For $C_d > 0.90$, N would have to be in error by more than ten per cent to cause an error of one per cent in C_{f_c}, so that great accuracy of N is not essential. It may also be noted by inspection of Fig. 13.2-6 that the variations of the profile would have little effect on the integrated value of N except where there is a relatively large region of the flow within ± 0.02 of $M = 1.0$.

The sonic test nozzles of Ref. [9] (see Fig. 13.2-2) provide an opportunity to apply the above analysis for comparison with experimental values. Using the values of the wall Mach number and the corresponding values of N, the experimental values of C_d are employed to predict C_{f_c} using Eq. (13.2-21). The predicted and experimental values of C_{f_c} are compared in Fig. 13.2-8. Since several experimental values of C_{f_c} fall below unity there is reason to believe that the experimental values of C_d are as much as 0.5 per cent low. Since C_d was used to calculate the experimental C_{f_c} (defined in terms of actual airflow) a comparison was also made on the basis of values of C_d

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

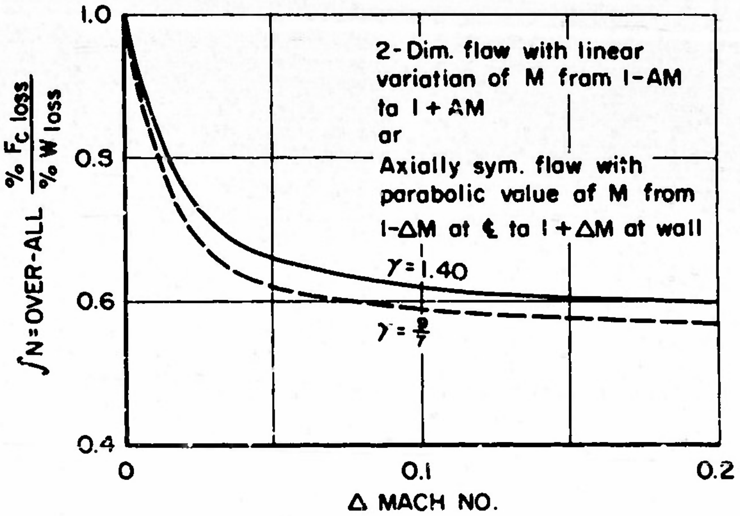


Fig. 13.2-7 OVER-ALL RATIO OF PERCENTAGE OF STREAM THRUST LOSS TO PERCENTAGE FLOW LOSS FOR TWO TYPES OF FLOW

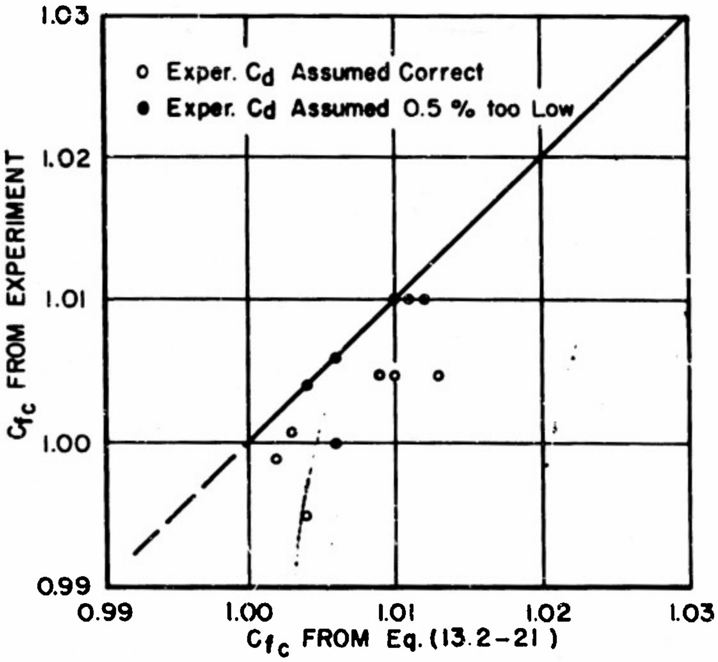


Fig. 13.2-8 COMPARISON OF EXPERIMENTAL AND THEORETICAL EFFECT OF WALL CURVATURE AT THE THROAT ON C_{f_e}

The data were obtained from Ref. [9].

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

which are 0.5 per cent higher than the test values. As Fig. 13.2-8 shows, the agreement between the theoretical and experimental values is good.

It should be noted that a profile with variation will have a lower stream thrust than a uniform M = 1.0 profile since the discharge decreases at a faster rate than C_{f_c} increases.

The above analysis makes it possible to predict the stream thrust loss from the discharge coefficient and the wall Mach number. Actually if one knew the wall Mach number one might also estimate C_d. Since it would also be necessary to assume a Mach number profile, use of the wall Mach number to obtain C_d is not believed satisfactory as shown in Fig. 13.2-5.

The effects of wall curvature at the throat are summarized as follows:

1. Considerable variation of the flow, Mach number, velocity and pressure normal to the mean flow direction can occur in a constriction with isentropic flow.
2. The variation is greatest when the wall curvature at the throat becomes small compared to the throat diameter.
3. When r/D_c is less than about 1.0 the discharge coefficient begins to fall rapidly.
4. Where losses are caused by Mach number profiles, the percentage stream thrust loss is about 60 per cent of the percentage discharge loss (except for M ±0.02 where the losses are about equal).
5. Thus the discharge coefficient can be used to estimate the thrust loss.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

The Effects of Wall Slope at the Throat

The convergent nozzles considered up to now have had parallel walls at the exit. The performance (C_d and C_{f_c}) of such nozzles is constant above a pressure ratio slightly above the critical pressure ratio. At high pressure ratios there is, of course, an expansion of the jet to ambient pressure beyond the nozzle exit. Distinctly apart from this effect is the effect found in some nozzles of jet contraction or formation of a vena contracta beyond the minimum geometrical area.

The presence of a slope on the walls at the minimum section gives rise to a radial momentum of the flow near the wall that causes a contraction of the jet. This effect, most pronounced in orifice flow, has been correlated with a theoretical "contraction coefficient" for orifice flow in Ref. [11]. Reference [12] shows the pressures and the jet contraction for an orifice. These are shown to be dependent on the constriction ratio. Although this same jet contraction effect shows up to a lesser degree in converging conical nozzles, there appears to be no theoretical equation for its prediction in this case. Reference [13] presents results of discharge and thrust tests on conical constrictions. Discharge coefficient increased with increasing pressure ratio and with decreasing conical angle between the walls. For small cone angles the discharge coefficient did not vary greatly with constriction ratio while for large cone angles the discharge coefficient increased as constriction ratio increased. Some of the results of Ref. [13] are shown in Fig. 13.2-9.

The distinctive feature of both the orifice and conical constriction is that no limiting mass flow is found at the critical pressure ratio as one-dimensional equations would indicate. It is indicated in Fig. 13.2-9 that the pressure ratio at which the discharge coefficient becomes approximately constant increases with cone angle.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

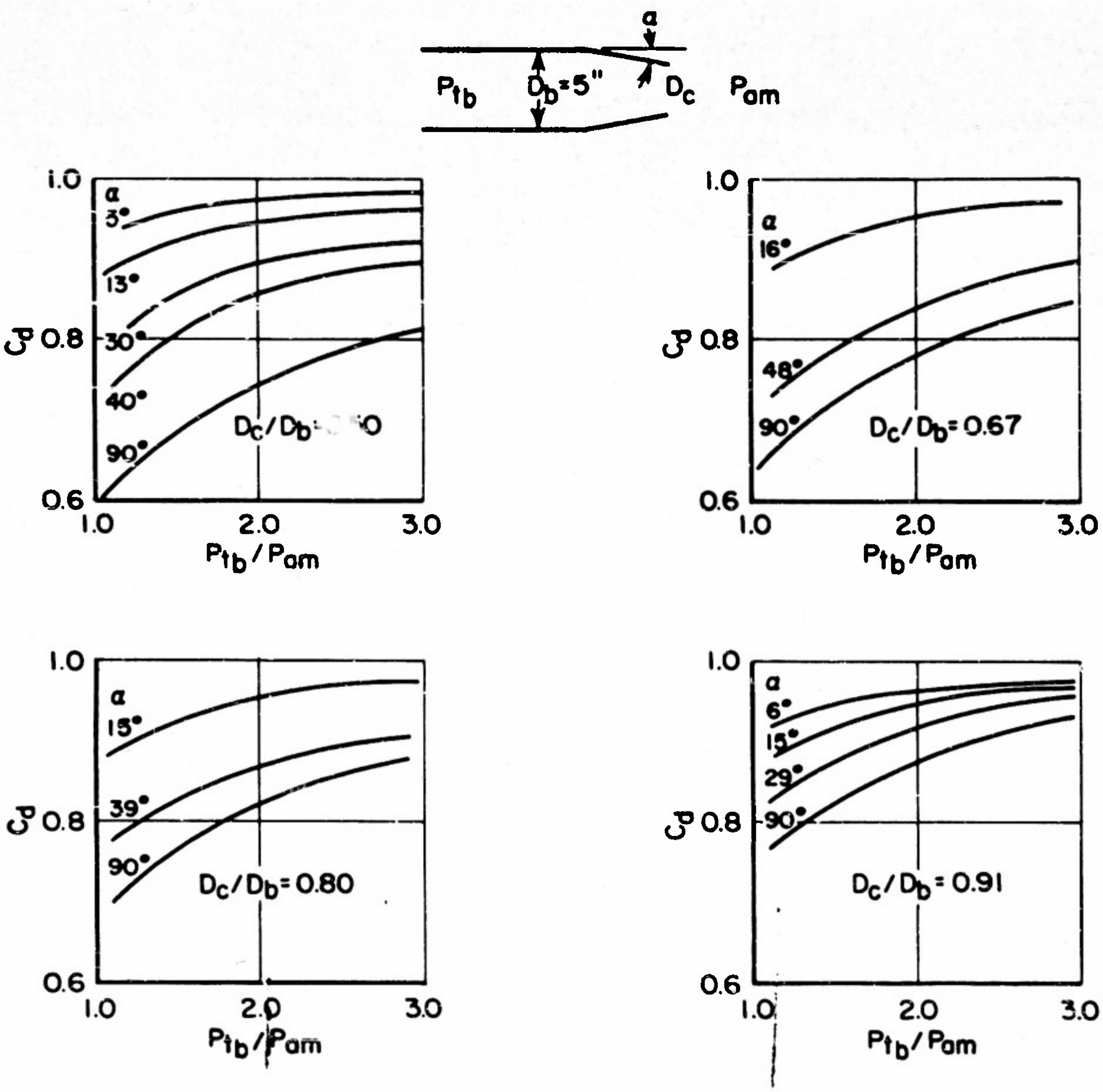


Fig. 13.2-9 DISCHARGE COEFFICIENTS FOR CONICAL CONSTRICTIONS

The data were obtained from Ref. [13].

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

For convergent nozzles which have a contraction of the jet beyond the minimum geometrical area (or throat) the ϕM at the geometrical throat is not expected to be unity. The following deviations determine the value of ϕM under such a condition.

Assuming the pressure on the contracted free surface of the jet to be equal to the pressure of the discharge region, the continuity and momentum equations would give the following results. Subscript "c" denotes the exit or minimum geometrical area of the nozzle and "c'" the minimum jet or vena contracta area.

$$F_{c'} = F_c - P_{am} (A_c - A_{c'})$$

$$W_a \; S_a \; (\phi M)_{c'} = W_a \; S_a \; (\phi M)_c - P_{am} \; A_{c'} \left(\frac{A_c}{A_{c'}} - 1 \right)$$

$$= W_a \; S_a \; (\phi M)_{c'} - \frac{W_a \sqrt{T_{t_c}}}{\mathring{m}_{c'}} \left(\frac{A_c}{A_{c'}} - 1 \right)$$

$$\frac{\phi M_c}{\phi M_{c'}} = 1 + \frac{\sqrt{T_{t_c}}}{S_a \; M_{c'} \; \mathring{m}_{c'}} \left(\frac{A_c}{A_{c'}} - 1 \right). \qquad (13.2\text{-}22)$$

Since $\phi M_{c'}$ at the flow minimum area should be unity or very nearly so, and since $S_a / \sqrt{T_{t_c}}$ equals 2.38 for cold flow or approximately 2.42 (1 + f/a) for hot flow, the ϕM ratio in Eq. (13.2-22) is principally a function of the jet contraction, $A_{c'}/A_c$ as shown in Fig. 13.2-10. Also shown in Fig. 13.2-10 is the experimental C_{f_c} for a portion of the data presented in Ref. [13]. It is assumed that C_d is equal to $A_{c'}/A_c$ for plotting purposes. Values for cone half angles from 5 to 90 degrees and nozzle constriction ratios (A_c/A_b) from 0.25 to 0.82 are plotted.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

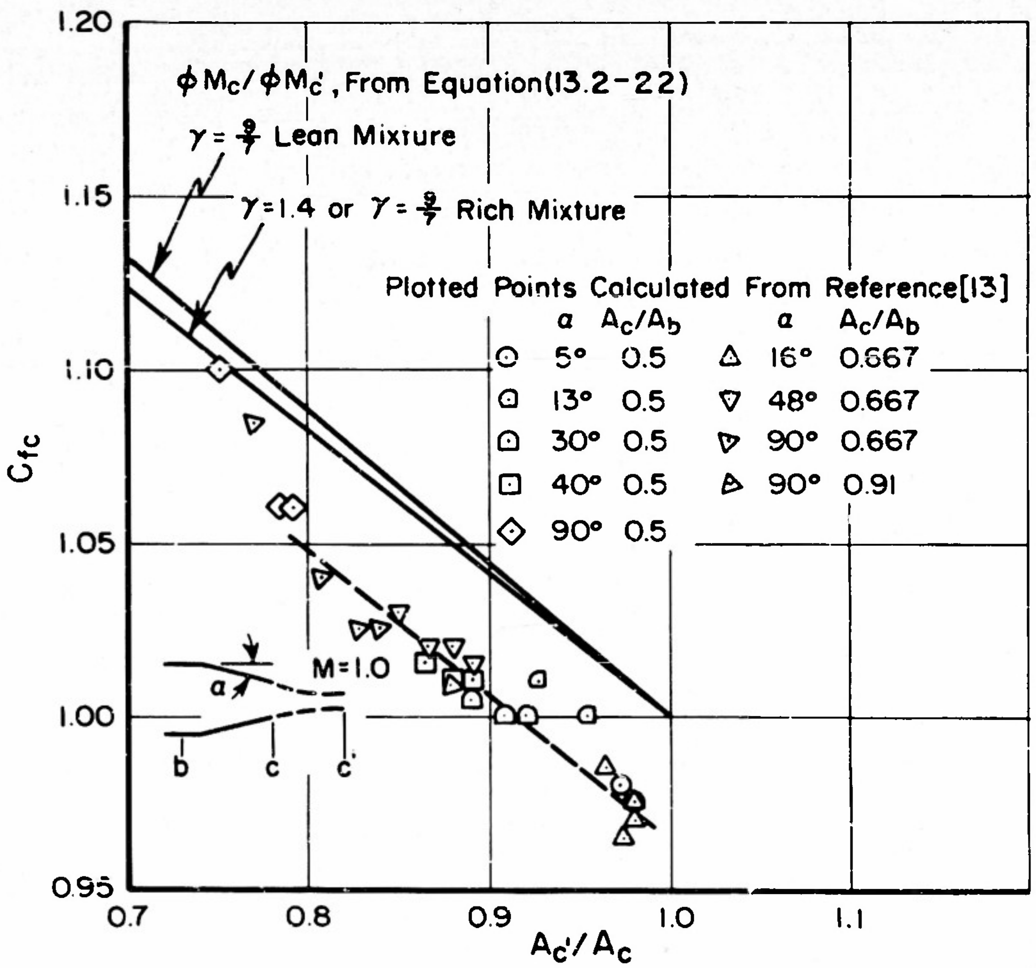


Fig. 13.2-10 EFFECT OF EXTERNAL JET CONTRACTION ON THE STREAM THRUST COEFFICIENT

CONFIDENTIAL

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

The plotted data are about 3.5 per cent below that which would be expected from Eq. (13.2-22).

Apparently (see Fig. 13.2-10) the primary variable controlling the stream thrust is the amount of jet contraction and not how that contraction was produced, since the data for a wide variety of constriction ratios and wall slopes all plot very nearly on a common line. The two points close to the curves for Eq. (13.2-22) are of doubtful validity in this comparison since for these points the nozzle was barely choked. At first glance, it would appear advantageous to use this type of exit constriction. It is true that the stream thrust exceeds that of an ideal nozzle having the same nozzle inlet airflow and total temperature. While C_{f_c} correctly describes the stream thrust of the flow it does not take into consideration that, for a nozzle with jet contraction, a larger exit area is required which will affect the atmospheric term in the C_t equation. The following example illustrates the true picture.

Assume a nozzle having jet contraction for which $D_c/D_b = 0.6$ and $\alpha = 90$ degrees. Assume the nozzle total pressure is 25 lb/in^2 absolute and the base or ambient pressure is 10 lb/in^2 absolute. A C_d of 0.79 is given in Fig. 13.2-5 and a C_{f_c} of 1.05 is given in Fig. 13.2-10. For a geometrical minimum area of 100 square inches the ideal airflow would be

$$W_{a_i} = P_t A (P\overset{\circ}{m}/P_t)_i / \sqrt{T_t} = 25\ (100)(0.5162)/\sqrt{T_t}$$

$$= 1290/\sqrt{T_t},$$

the actual airflow would be

$$W_a = (0.79)(1290/\sqrt{T_t}) = 1020/\sqrt{T_t},$$

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C., SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

and the ideal stream thrust would be

$$F_{c_i} = P_t A(f/P_t)_{c_i} = 25\ (100)(1.253) - 3130 \text{ pounds}$$

$$= W_a\ S_a\ \phi M = (1290)/\ \sqrt{T_t})(2.42\ \ \sqrt{T_t})(1.0) = 3130 \text{ pounds.}$$

The actual throat stream thrust from Eq. (13.2-12) would be

$$F_c = 3130\ (1.05)(0.79) = 2600 \text{ pounds.}$$

Consider a nozzle having parallel walls at the exit with negligible losses, i.e., $C_d = C_{f_c} = 1.0$. Assume the same inlet total pressure and temperature and the same ambient pressure. The throat area would be

$$A = \frac{1020}{25\ (0.5162)} = 79 \text{ square inches}$$

and the stream thrust would be

$$F_c = 25\ (79)(1.253) = 2470 \text{ pounds.}$$

To compare missile performance with these two nozzles, an atmospheric term, $P_{am}(A_e - A_o)$ must be subtracted. With the inlet area, A_o and the ambient or base pressure, P_{am} (the same in both cases), the quantity to be subtracted is larger for the nozzle having jet contraction by the following amount:

$$P_{am}(100 - 79) = 10\ (21) = 210 \text{ pounds.}$$

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

Accounting for this difference, the "net" stream thrust of the nozzle having jet contraction is 2390 pounds as compared to 2470 pounds for the nozzle having no jet contraction. If the test data had agreed with Eq. (13.2-22) this example would have given identical "net" stream thrusts for the two nozzles. The reason is that Eq. (13.2-22) was based on ambient pressure in the contracted jet.

One other factor should be noted in connection with Fig. 13.2-10. The test values of C_{f_c} should not have been below 1.0 so that some systematic error may have been present. Although no mention is made in Ref. [13] of a reduced pressure on the outside of the nozzles, it is possible that such a pressure could have affected the results.

The important conclusion of this discussion is that there is no thrust advantage to be gained from wall slope at the throat. On the other hand, if the possibility of a systematic error in the data shown in Fig. 13.2-10 is considered, no serious thrust penalty may result from using wall slope at the throat. There is, however, considerable variation of the discharge coefficient (Fig. 13.2-9) so that calculation of combustor or missile performance would be considerably more cumbersome than with a constant C_d.

Effects of Burning in the Nozzle

Since accurate pressure and temperature measurements at the exit of a ramjet combustor are very difficult to make, most ramjet exit-nozzle performance testing is done with cold flow. As there is a negligible total temperature change with cold-flow tests, the results are directly applicable to burning performance where the burning in the exit nozzle is negligibly small.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

The air specific impulse (S_a) at the nozzle inlet is very difficult to measure and is not important except in establishing the degree of burning in the exit nozzle. The throat stream-thrust coefficient is therefore based upon the S_a at the throat. Inspection of Eq. (13.2-5) shows that C_{f_c} would not be affected by burning in the nozzle as long as the actual airflow and the throat S_a are used. The actual S_a at the throat is derived from a theoretical value and an assumed or experimental combustion efficiency. The performance of most ramjet combustors is evaluated at the throat of a sonic nozzle as there seems to be little point in evaluating a combustion efficiency for the combustor and exit nozzle separately.

The amount of total temperature change occurring as the gases pass through the exit nozzle will depend on the degree of completeness of combustion at the nozzle entrance, the rate of reaction, and the time spent in the exit nozzle. Present theory and experiment do not allow much more than a qualitative discussion of the total temperature change. Full-scale combustor tests with various convergent exit nozzles have shown that, for sea-level or high-pressure tests (where the completeness of combustion and reaction rates are comparatively high), there is practically no change in S_a at the exit of a sonic nozzle 0.2 D_c long (nozzle "B") (length = 20 per cent of throat diameter) when compared to one 1.4 D_c long (nozzle "C"). In low pressure altitude tests where the completeness of combustion and reaction rates are lower, the use of the longer exit nozzle provided a substantial increase in S_a at the exit, particularly at high equivalence ratio levels (where the completeness of combustion at the nozzle entrance was low).

Comparison of these performance gains with those from tailpipe extensions ahead of the exit nozzle indicated that the performance gain could be roughly correlated with the increased time that the gases spent in the engine. Although the reaction

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

rate in the exit nozzle is reduced by the lowered pressures, the time increase will provide a first approximation of the total temperature increase. In order to make reasonable estimates it would be necessary to have the relative performance with two exit nozzles or with various tailpipe lengths.

The effect of the exit configuration on the combustion efficiency of the Talos combustor is shown very roughly in Fig. 13.2-11. Nozzle "C" has given practically the same performance as nozzle "B" with a 12-inch tailpipe extension and has practically the same residence time so that these two configurations are considered equal and form the basis of comparison for the other configurations. Comparative residence times of each combustor nozzle has been calculated for a particular S_a. The residence times of the shorter configurations can be expressed as a fraction of the total residence time of the combustor long sonic-nozzle configuration as long as the S_a (or $S_a/\sqrt{T_{t_o}}$) of the two configurations is the same. The comparative combustion efficiency values were obtained by comparing the efficiencies of a short-exit configuration with either nozzle "C" or nozzle "B" with the 12-inch tailpipe at the same $S_a/\sqrt{T_{t_o}}$.

There was considerable scatter as well as regions where the long configuration gave an $S_a/\sqrt{T_{t_o}}$ which the short configuration could not produce. Therefore, the curves show only the general trends.

It is apparent from Fig. 13.2-11 that the exit configuration affects the combustion efficiency; the effect becoming more pronounced as air flow is reduced (increase in altitude).

The use of a 65 per cent exit nozzle in place of a 70 per cent nozzle would reduce the combustor Mach number. Since the nozzles represent only four to twenty per cent of the total residence time the effect of a reduction of the exit constriction would be to reduce the horizontal width of Fig. 13.2-11 as well

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

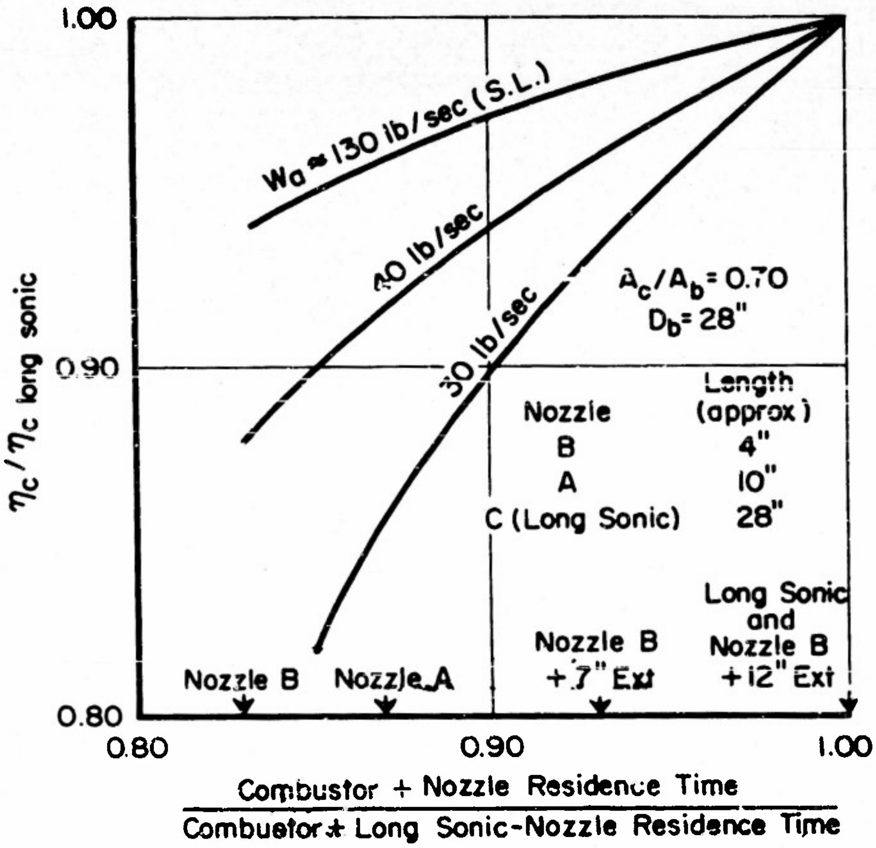


Fig. 13.2-11 APPROXIMATE RELATION OF COMBUSTOR AND NOZZLE RESIDENCE TIME AND COMBUSTION EFFICIENCY--TALOS MODEL G

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

as to increase the values of efficiency because of the longer residence time. Consequently, the effect of the exit configuration becomes less as the constriction ratio decreases. Tests with a 65 per cent exit nozzle qualitatively confirm this conclusion.

The rather large effects shown in Fig. 13.2-1. result from the unusually short length of the Talos combustor.

It is clear from the parameters which define the amount of burning in the exit nozzle that the total temperature change is peculiar to a particular burner-exit nozzle combination and would be quite difficult to define in general. Since the flow is actually controlled by the conditions at the throat, a temperature rise from the inlet to the throat will reduce the airflow for the same upstream total temperature and, except for total pressure effects, would reduce the C_d by a ratio equal to the square root of the throat to inlet total temperature. Figure 13.1-4, which represents burning in a straight pipe, indicates that it is advantageous to burn in as large a pipe as possible from the standpoint of total pressure loss. While the information in Fig. 13.1-4 is not applicable to a duct of changing area it is probable that there would be a total pressure loss which would give a still lower C_d based on the upstream total pressure. Disregarding flow profile effects, the ratio of the actual nozzle inlet Mach number to the isentropic, one-dimensional value is proportional to C_d so that burning in the nozzle would lower the nozzle inlet Mach number.

It must be emphasized, however, that these effects are of little interest where the S_a and total pressure recovery of the combustor-convergent nozzle combination have been determined by experiment. The designer of a new powerplant for which no test data are available must rely on estimates of the amount of burning and its effect on S_a and pressure recovery as well as on the discharge.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

Since data on the measured total pressure loss in exit nozzles with burning are not plentiful, Fig. 13.2-12 is included. Inasmuch as the total pressure calculated from thrust does not agree with rake data, the only method believed reliable for determination of the nozzle-inlet total pressure ratio is the use of nozzle inlet and exit pressure rakes. The curves shown in Fig. 13.2-12 represent average rake data with the scatter being about ± 2 per cent for nozzle "B" and about ± 1 per cent for nozzle "A".

In the section on friction it was shown, that for nozzles as short as these, the total pressure loss would be of the order of 1 per cent or less. The fact that these nozzles attain values of total pressure ratio as high as 98 or 99 per cent seems to confirm the small friction loss.

The large variation of the loss with M_2 is believed to be caused primarily by burning in the nozzle since M_2 is principally dependent on combustor heat release for a given burner. A high M_2 indicates a small heat release which would be quenched by the time the flow reached the exit nozzle, so that one would expect only slight burning in the exit nozzle. A low M_2 indicates a high heat release which might be indicative of appreciable burning in the nozzle.

Extremities of the range of the total pressure loss can be estimated by assuming that the burning takes place just ahead of the nozzle in a constant area duct or that it takes place in a straight duct whose area equals the throat area. In the first case burning raises the Mach number to the isentropic value corresponding to the constriction ratio, while in the latter case burning raises the Mach number to 1.0. Since the airflow and stream thrust are assumed constant, the Mach number before burning can be obtained directly. Letting subscript 1 denote conditions before burning in the nozzle and subscript 2 denote conditions after burning, and assuming a total temperature ratio

$$\phi M_1 = \frac{S_{a_2}}{S_{a_1}} \phi M_2 = \sqrt{\frac{T_{t_2}}{T_{t_1}}} \phi M_2 \qquad (13.2\text{-}23)$$

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

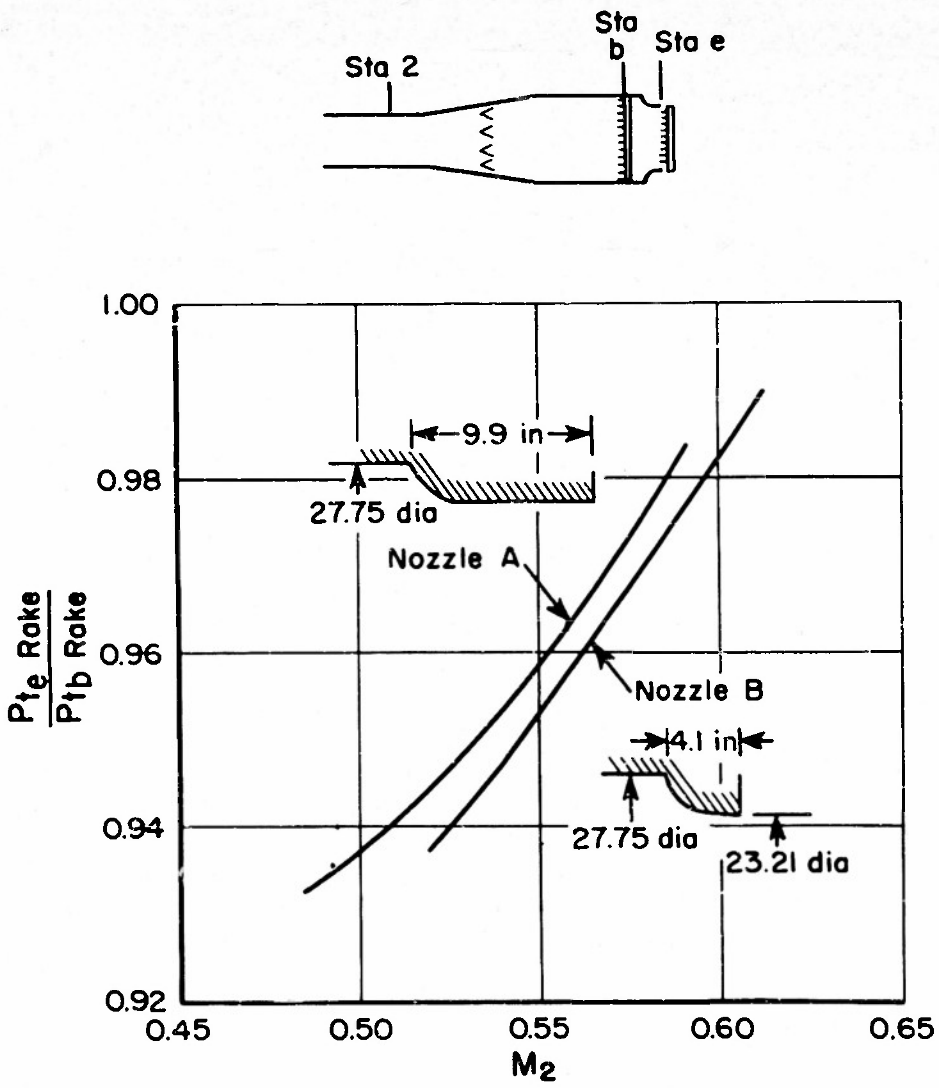


Fig. 13.2-12 MEASURED TOTAL PRESSURE LOSS ACROSS TWO FULL-SCALE TALOS SONIC EXIT NOZZLES

These data were obtained with burning taking place at an airflow of 40 lb/sec.

CONFIDENTIAL

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

With both Mach numbers known and the area constant, the total pressure loss can be calculated

$$\frac{P_{t_2}}{P_{t_1}} = \frac{(f/P_t)_1}{(f/P_t)_2} \qquad (13.2\text{-}24)$$

Calculated values of the temperature ratio required to produce a given total pressure loss are tabulated below:

P_{t_c}/P_{t_b}	T_{t_c}/T_b Required: Burning Ahead of Nozzle	T_{t_c}/T_b Required: Burning in Throat
1.00	1.00	1.00
0.98	1.12	1.04
0.96	1.30	1.11
0.94	1.60	1.20

Temperature rises as high as 17 per cent in the exit nozzle have been measured near the peak S_a by means of a total temperature probe. This would be sufficient to cause the measured loss if most of the burning occurred in the throat.

Equations (13.2-23) and (13.2-24) may also be used to obtain an integrated total pressure loss resulting from burning in any portion of the exit nozzle. With the rate of burning defined or known, a step-by-step numerical integration can be started at the downstream end of the minimum geometrical area where the one-dimensional Mach number will be 1.0. By working upstream with burning in short ducts of constant area alternated with isentropic area changes, an integrated value of P_t loss can be obtained to any degree desired by the use of a sufficient number of increments. The accuracy, of course, will depend on the validity of the assumed burning distribution.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

13.3 PERFORMANCE OF SUPERSONIC EXIT DIVERGENCES

In the convergent section of a convergent-divergent nozzle, the flow is, for the most part, subsonic in velocity, Since most convergent-divergent nozzles use a reasonably rounded inlet, jet contraction effects are usually absent. Downstream of the throat in the divergent section the flow is mostly supersonic if sufficient pressure ratio is applied across the nozzle. In this section the flow in the divergent section is discussed. This discussion applies not only to convergent-divergent nozzles but to exit divergence from a straight pipe as well, however, the entry conditions to the divergence may differ slightly.

As described in the section on the relation of the exit nozzle to the complete ramjet, the use of a supersonic divergent section can provide gains in thrust coefficient and fuel specific impulse over a sonic exit nozzle. It was also shown that the amount of gain was not only affected by the flight Mach number but also by exit stream thrust ratio, C_{f_e}. It was shown in Fig. 13.1-3 that there might be regions where the exit divergence would produce a lower C_t and I_f than a sonic nozzle. A low exit stream thrust coefficient was shown to reduce the available C_t and I_f.

In order to judge the amount of gain or loss from a supersonic diverging section over a sonic exit nozzle, some knowledge of the performance of real flows in a divergent section is necessary.

One-Dimensional Relations

The flow in a divergent section is governed by the pressure ratio across it. For inlet total pressures very nearly equal to the exit ambient pressure, the flow is subsonic.

CONFIDENTIAL

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

One-dimensional relations for an isentropic flow show that, for a given divergent exit to throat area ratio, A_e/A_c, there is a particular pressure ratio for which the exit stream static pressure exactly equals the ambient pressure of the discharge region. This pressure ratio is called the design pressure ratio which is most easily expressed in terms of the exit Mach number. For one-dimensional isentropic flow, the area ratio and pressure ratio in terms of the exit Mach number are as follows. Letting "c" denote the throat and "e" the exit, and using a throat Mach number of 1.00

$$\frac{A_e}{A_c} = \frac{1}{M}\left[\frac{2 + (\gamma - 1)\,M^2}{\gamma + 1}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \tag{13.3-1}$$

$$\frac{P_{t_c}}{P_e} = \left[1 + \frac{\gamma - 1}{2}\,M^2\right]^{\frac{\gamma}{\gamma-1}} \tag{13.3-2}$$

Suitable graphs and tables of these and other Mach number functions are available in Refs. [14] and [15], and are also given in the Appendix.

If the exit ambient pressure is lower than that required for the design pressure ratio (inlet total pressure constant), the flow in the divergence is exactly the same as the flow at the design pressure ratio. There is an expansion to the ambient pressure beyond the exit which gives rise to the expression that the nozzle, under these conditions, is underexpanded. If the exit ambient pressure is higher than that required for the design pressure ratio, there will be a nonisentropic process by means of which the static pressure rises to the value in the discharge region. For one-dimensional analysis this must be a normal shock since oblique shocks cannot occur in one-dimensional

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

flow. For real flows, however, the pressure rise has been found to occur through oblique shocks with separation of the flow from the nozzle walls. (See Figs. 13.3-1 and 13.3-2.) A nozzle operating with a discharge pressure higher than that for the design pressure ratio is referred to as overexpanded.

The exit stream thrust for a supersonic divergence may be expressed by one-dimensional isentropic equations for nozzle pressure ratios (P_{t_b}/P_e) near or above the design pressure ratio.

$$\begin{aligned} F_e &= m V_e + P_e A_e \\ &= P_{t_e} A_e (f/P_t)_e \\ &= W_a S_a \phi M_e \end{aligned} \qquad (13.3\text{-}3)$$

For one-dimensional isentropic flows, the various forms of the above equation will yield identical values of F_e. It is apparent from Eq. (13.3-3) that, for a given airflow and S_a (controlled by conditions upstream of the throat), the maximum practical value of F_e will be attained if the exit area is made equal to the maximum missile frontal area, since ϕM_e increases as the exit Mach number and the exit area increase. There would probably be little gain in thrust coefficient or fuel specific impulse by expanding to an area larger than the maximum frontal area.

Expansion of the exit is favorable to increased F_e, and also reduces boattail drag and consequently explains the gains in thrust coefficient and fuel specific impulse over a sonic nozzle.

The analysis of the exit divergence with considerable over-expansion does not lend itself to one-dimensional treatment and is discussed separately in a later section.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

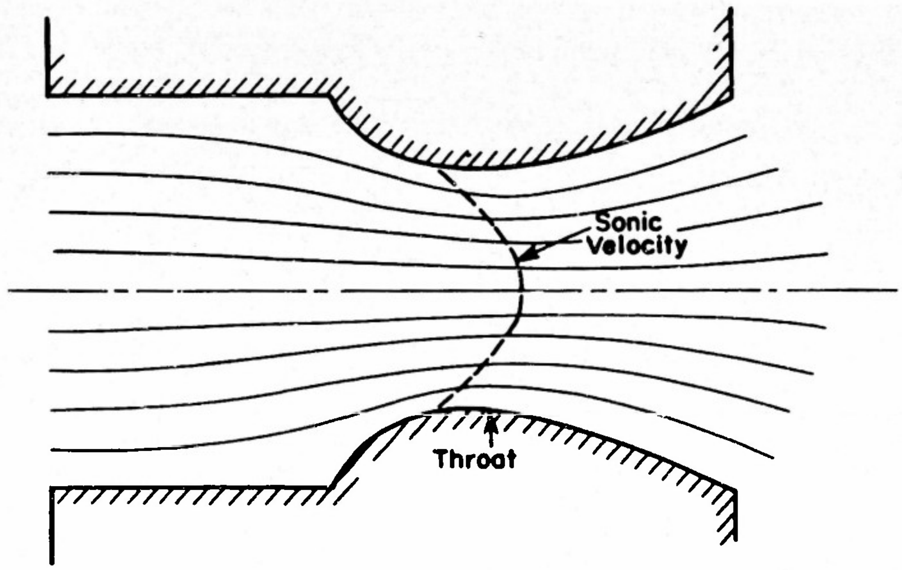


Fig. 13.3-1 GENERAL NATURE OF THE FLOW IN A CONVERGENT-DIVERGENT NOZZLE NEAR THE DESIGN PRESSURE RATIO

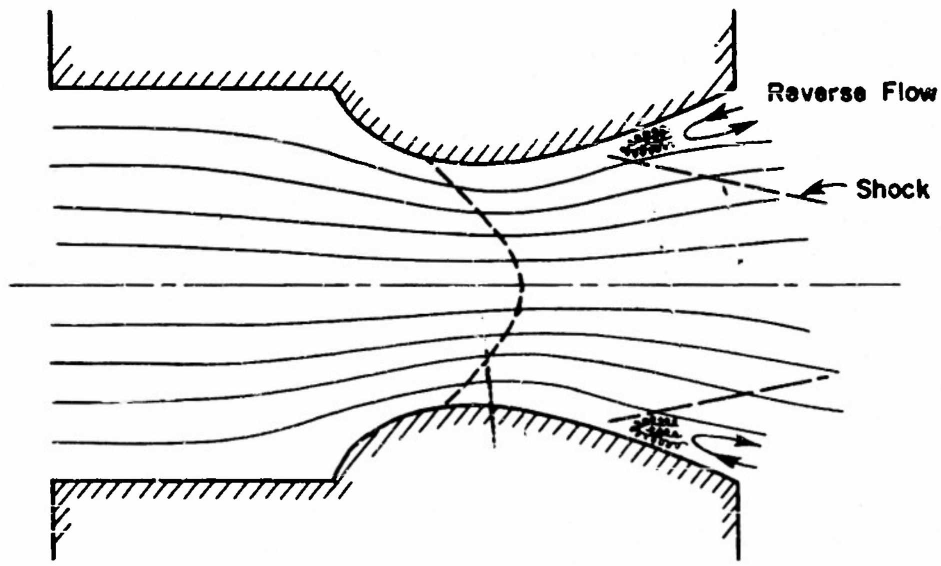


Fig. 13.3-2 SEPARATION OF THE FLOW IN AN OVEREXPANDED NOZZLE--EXIT PRESSURE TOO HIGH

CONFIDENTIAL

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

Definition of Coefficients

Since the coefficient of discharge is controlled almost entirely by the portion of the nozzle upstream of the throat, the only item of interest in the divergent section is the ratio of actual stream thrust obtained to that which one would expect from one-dimensional isentropic flow. Tests reported in Ref. [9] have shown that the coefficient of discharge of a given inlet is reduced by the addition of a supersonic section. For practical ramjet nozzles this effect will be less than about one per cent. The exit stream thrust coefficient is defined as follows

$$C_{f_e} = \frac{F_e}{W_a \, (S_a)_c \, \phi M_{e_i}} = \frac{F_e}{C_d \left(\frac{S_{a_c}}{S_{a_e}} \right) F_{e_i}} \qquad (13.3\text{-}4)$$

The subscript "e" denotes the exit, "i" denotes ideal, F_e is the actual stream thrust and W_a is the actual airflow. The throat S_a is again used for purposes of definition since the combustion efficiency is most often measured at the throat. In cases where the combustion efficiency at the exit is known or where there is no change in S_a from the throat to the exit, Eq. (13.3-4) reduces to

$$C_{f_e} = \frac{\phi M_e}{\phi M_{e_i}} \left(\frac{S_{a_e}}{S_{a_c}} \right) = \frac{\phi M_e}{\phi M_{e_i}} \qquad (13.3\text{-}5)$$

where ϕM_e is based on the actual airflow.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

Divergence with Uniform Flow at the Exit

The ideal divergence would be shockless, frictionless, and would have the entire flow at the exit parallel to the axis. This condition is approached in supersonic nozzles used in wind tunnel work where the primary objective is a uniform shock-free flow. The length of these nozzles is prohibitive for almost all ramjet powerplant applications. It has been found in Refs. [16] and [17] that the shortest possible length of supersonic divergence which has uniform flow at the exit is obtained with a sharp corner at the throat. It may be noted from Fig. 13.3-3 that even these shortest lengths are impractical for ramjet application for both two-dimensional and axially symmetric nozzles. This type of nozzle has its wall profile shaped to turn the flow parallel to the axis, the wall shape being determined by the method of characteristics. Observed pressures on the walls agree almost exactly with a characteristic solution. Consequently, the experimental stream thrust deviation from the ideal would arise mostly from friction.

Tests have been made at the San Diego Propulsion Laboratory on two nozzles of this type to determine the exit stream thrust coefficient. The throat diameter was 1.94 inches, the exit diameter 2.23 inches, and the divergence length was 3.50 inches. Using air at ambient temperature and at total pressures from two to five atmospheres (discharge to ambient pressure) the mean value of C_{f_e} was 1.00 which is within the experimental error of about one per cent. Assuming pipe friction, the effects of friction in the divergent section of these tests would have reduced C_{f_e} by about 0.5 per cent, an amount within the experimental error. One of these nozzles had an inlet similar to nozzle "B" of Fig. 13.2-2 and the other was similar to nozzle "E". There was no consistent difference between the C_{f_e} values using

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

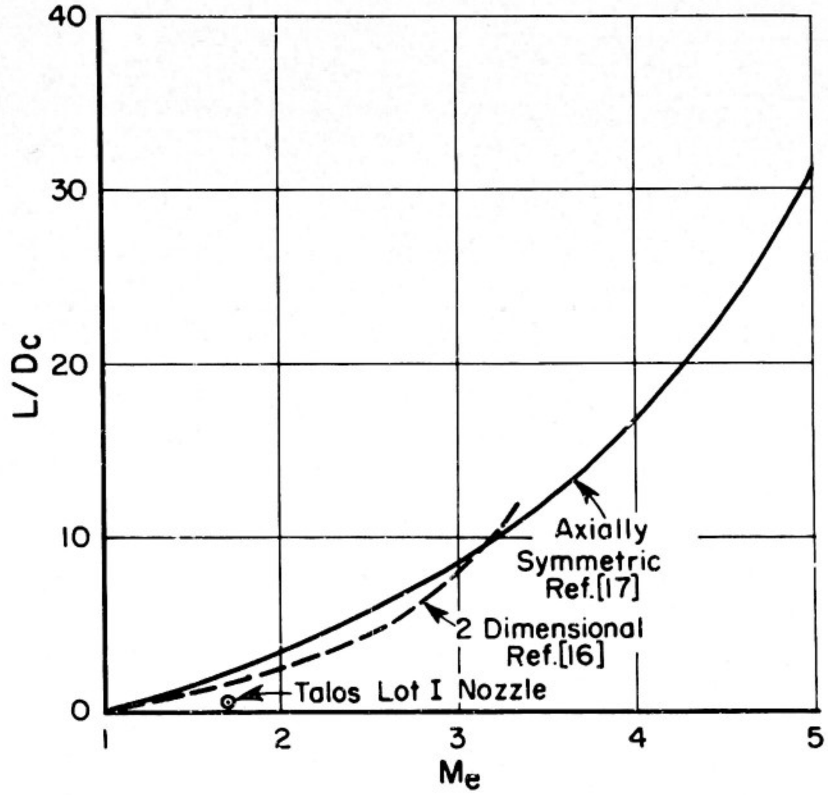


Fig. 13.3-3 MINIMUM LENGTH OF DIVERGENT SECTION FOR UNIFORM FLOW AT THE EXIT--SHARP CORNER AT THE THROAT

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

these two inlets, the results being within 0.2 to 0.7 per cent of each other. Separation effects appeared in these nozzles between pressure ratios (P_{t_b}/P_{am}) of 2.0 to 2.3. The design pressure ratio was about 4.8.

Since the Reynolds number of ramjet exit nozzles will be high, the main objection to using long divergences is not so much the loss resulting from friction but rather the weight and length requirements.

Practical Ramjet Divergence Losses

Since divergences with uniform flow at the exit are so bulky, it is desirable to use a shorter divergence. The most common shorter divergent section is the simple conical divergence usually connected to the throat by means of a smooth curve or arc. The principal reasons for this choice are the relatively low loss of thrust and the ease of fabrication. Other divergent forms utilizing various curved profiles or special forms may also be used.

There are several reasons for a loss of C_{f_e} using a short diverging section, the principal ones being

1. friction at the wall,
2. exit flow not parallel to axis, and
3. variations of Mach number at a section.

The effects of friction are most important for long divergences, particularly those having small angles of divergence. As discussed more fully later, the flow at the exit is, to a first approximation, conical in axially-symmetric conical divergences so that lack of parallelism of the flow and the axis is most important for high divergence angles. Even though the exit flow may be essentially conical, the Mach number on a

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

spherical surface may not be constant, indicating that Mach number variations should be considered in an analysis.

A loss not listed above is the loss due to mixing of portions of the flow having different properties, particularly temperature. Reference [19] describes tests of a small nozzle having a 69 per cent constriction with both uniform and nonuniform temperature profiles at the exit. Profiles were obtained by placing a small burner close to the nozzle inlet. For ratios of maximum to minimum total temperature of about 1.8 ($T_{t_{max}}/T_{t_{avg}} = 1.4$) it was concluded in Ref. [19] that there was no change in discharge or exit stream thrust coefficient from the values obtained with uniform flow. Since, from a mixing standpoint, one might expect a change, it is probable that any changes which did occur were masked by experimental error which was stated to be about one per cent. Because of present inadequacy of mixing theory and the small discrepancy indicated by Ref. [19], no further consideration will be given to mixing-effect losses.

The Effects of Wall Friction

Friction effects in the divergent section are most important for low divergence angles. A good approximation of the friction thrust loss for low divergence angles can be obtained by using pipe friction equations considering the divergence to be made up of a series of cylindrical segments,

$$\frac{\Delta F}{F_c} = \frac{(\Delta P)A}{F_c} = \frac{4f(L/D)qA}{P_{t_c} A_c (f/P_t)_c} \qquad (13.3\text{-}6)$$

Because of the high Reynolds numbers and the small magnitude of the friction losses, great accuracy is not required. For a first approximation, then, the friction total pressure loss can

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

be neglected. The total frictional effect summed up to any desired area ratio and expressed in terms of the ideal exit stream thrust at that area ratio is as follows

$$\frac{\Sigma(\Delta F)}{F_{e_i}} = \frac{\Sigma(\Delta F)}{F_c \; \phi M_{e_i}} = \frac{\Sigma\, 4f\,(L/D)\,(q/P_t)\,(A/A_c)}{(f/P_t)_c \; \phi M_{e_i}} \; . \qquad (13.3\text{-}7)$$

Neglecting the discharge coefficient and burning in the divergent section

$$C_{f_e} = \frac{F_e}{C_d(S_{a_c}/S_{a_e})F_{e_i}} = \frac{F_e}{F_{e_i}} = 1 - \frac{\Sigma(\Delta F)}{F_{e_i}} \; . \qquad (13.3\text{-}8)$$

If unidimensional Mach numbers are used, the Reynolds number at any station relative to that at the throat equals $D_c\mu_c/D\mu$ where μ is the viscosity. Variation of the viscosity ratio is small since viscosity is roughly inversely proportional to the square root of the temperature.

The effects of friction on C_{f_e} are shown in Fig. 13.3-4 for various area ratios and a throat Reynolds number of 10^6 Unidimensional values of Mach number were used.

It is apparent that for the high values of Reynolds numbers found in most ramjet applications, the frictional effects, even for large area ratios, are quite small for total divergence angles above 30 degrees.

Conical Flow Thrust Loss

Approximate conical flow is obtained in a conical diverging section particularly where a smooth transition is used at the throat.

CONFIDENTIAL

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

The loss of thrust is described in Ref. [18] by defining the ratio of the average exit velocity to the velocity in a jet parallel to the nozzle axis. From Ref. [18]

$$\phi_\beta = \cos^2 \left(\frac{\beta}{4}\right) , \qquad (13.3\text{-}9)$$

where β is the total divergence angle in radians. Conversion to the exit stream thrust coefficient can be made using the mV to F_e relationship at the design pressure ratio, since only the mV portion of the stream thrust is affected by the reduction in the average velocity. Again neglecting the discharge coefficient and burning in the divergence

$$C_{f_e} = 1 - \frac{mV}{F_e} (1 - \phi_\beta)$$

$$= 1 - \frac{(f/P)_e - 1}{(f/P)_e} (1 - \phi_\beta) . \qquad (13.3\text{-}10)$$

The values of C_{f_e} arising from exit flow angularity with the axis will thus vary slightly with exit to throat area ratio as shown in Fig. 13.3-5.

In Fig. 13.3-5 is shown a comparison of the values of C_{f_e} obtained from Eqs. (13.3-8) and (13.3-10) as compared with numerous experimental data from Refs. [9,10,18,20, and 21]. The variously defined values of thrust efficiency from these references have been converted to C_{f_e} for presentation in Fig. 13.3-5.

Hengartner and Werwerka [18] describe tests on a series of supersonic nozzles having a 7.5 mm diameter throat, expansion ratios (A_e/A_c) of about 2.1, total divergence angles from 5 to 60 degrees, and inlet total pressures from 3 to 14 atmospheres (discharge to atmospheric pressure). The inlet total temperatures are not stated.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

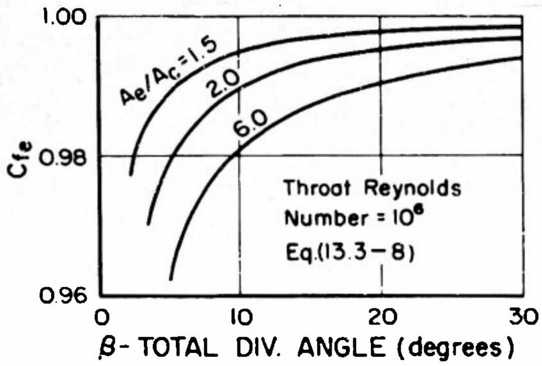


Fig. 13.3-4 EFFECT OF FRICTION ON EXIT STREAM THRUST COEFFICIENT

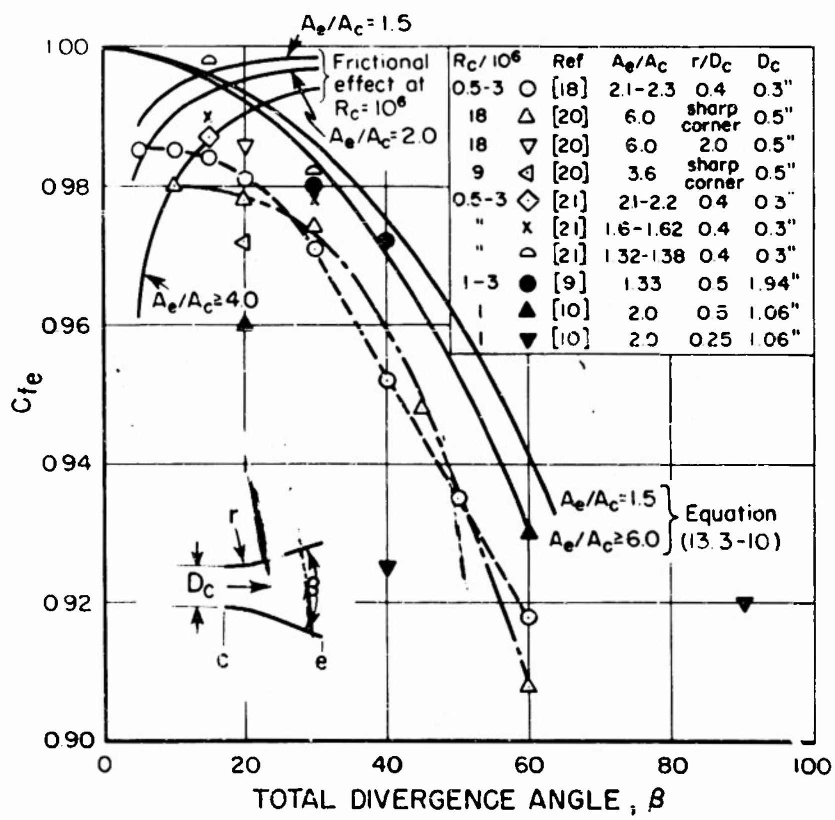


Fig. 13.3-5 THE EFFECT OF DIVERGENCE ANGLE AND FRICTION ON THE EXIT STREAM THRUST COEFFICIENT FOR CONICAL DIVERGENCES

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

The most favorable angle of divergence of these nozzles was about 15 degrees of the total angle. Extension of the model test data to full scale indicated that, since friction would be relatively decreased in the large nozzles, a slightly smaller angle would be optimum for full scale. An optimum of about 12 degrees was indicated for a 20-inch throat diameter with hot gas flow. Test data from Ref.[18] are shown in Fig. 13.3-5. The C_{f_e} loss increases rapidly beyond about 20 degrees total divergence angle. Although these nozzles were quite small the throat Reynolds number is estimated to be about 5×10^5 to 3×10^6. The data indicate that the values of C_{f_e} would be constant over a rather wide nozzle pressure ratio range but that overexpansion beyond about twice the design exit pressure causes separation with consequent changes in the exit stream thrust coefficient.

Fraser, Connor, and Coulter [20] describe the tests made to determine the thrust losses of a series of convergent-divergent nozzles having a conical divergence. These nozzles had a well formed inlet and a sharp corner at the throat. The nominal throat diameter was 0.5 inch and the exit area ratio (A_e/A_c) was 6.0 with total divergence angles from 10 to 60 degrees. These nozzles were tested at the design pressure ratio of about 63.5 atmospheres with discharge to the atmosphere. The throat Reynolds number is estimated to be about 1.8×10^7. These data are also plotted in Fig. 13.3-5. Since the coefficient of discharge was not given, it was assumed from the thrust ratio of a well rounded sonic nozzle (No. 30), to be 0.975 for all the nozzles having the same inlet. Again there is a pronounced drop in C_{f_e}, but this time at about 30 degrees total divergence angle.

A direct comparison [20] of a sharp cornered nozzle with a nozzle having a rounded wall profile at the throat ($r/D_c = 2.0$) indicated that the rounded throat gave a C_{f_e} of about one per cent higher than that with a sharp cornered throat. The exit

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

divergence total angle of 20 degrees and exit to throat area ratio of 6.0 was the same for both nozzles.

Another comparison [20] was made in which a sharp cornered nozzle having an exit to throat area ratio of 6.0 was compared to a nozzle having an exit area to throat area ratio of 3.6, both with a total divergence angle of 20 degrees. Comparison at their design pressure ratios shows that the C_{f_e} for the area ratio of 3.6 is about 0.6 per cent lower than for the area ratio of 6.0.

Curves in Refs. [18] and [20] show the increased influence of friction at the low divergence angles.

Hengartner [21] describes tests on nozzles similar to those shown in Ref. [18] except that the exit to throat area ratio was varied. Total divergence angles of 15 and 30 degrees were investigated. The results, in terms of C_{f_e}, are plotted in Fig. 13.3-5. The frictional effect is small but is definitely more pronounced at a divergence angle of 15 than at 30 degrees.

Tests were made at the Convair Propulsion Laboratory [9] on nozzles having total conical divergence angles of 30 to 40 degrees, a throat diameter of 1.94 inches, an exit to throat area ratio of 1.33, and wall curvatures at the throat of 0.5 D_c. Air at two to five atmospheres and ambient total temperature was used with discharge to ambient pressure. The results are also plotted in Fig. 13.3-5. The throat Reynolds number of these tests was from 1×10^6 to 3×10^6

Palmer and Bennett [10] show the results of tests on four supersonic nozzles having conical divergences. The exit to throat area ratio was 2.0, the throat diameter was 1.06 inches, and the throat curvatures were 0.5 and 0.25 D_c. The throat Reynolds number was approximately 10^6. The results at the design pressure ratio are shown in Fig. 13.3-5.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

The important conclusions to be obtained from Fig. 13.3-5 are as follows:

1. The exit stream-thrust coefficient for total conical divergence angles less than 20 degrees is controlled principally by friction and will consequently vary with nozzle-wall surface area, Reynolds number, and the finish of the wall surface.
2. For exit conical-divergence angles from about 20 to 60 degrees, there is an appreciable reduction of the exit stream thrust coefficient which is caused largely by the velocity component loss.
3. The best compromise for a conical divergence would probably be with an exit total divergence angle of about 20 to 30 degrees for which the exit stream thrust coefficient would be about 0.97 to 0.98.

Use of the Method of Characteristics

Although the assumption of conical flow gives values of C_{f_e} which are reasonably close to experimental values, such an assumption may not be valid, particularly near the throat.

The surfaces of constant Mach number in one-dimensional flow are planes normal to the axis, the Mach number being determined by the flow area relative to the throat area. For conical flow the surfaces of constant Mach number are spherical. If sections are taken parallel to the nozzle axis, the plane surfaces appear as straight lines and the spherical surfaces as arcs of circles. These are illustrated in Fig. 13.3-6, together with Mach number profiles determined by the method presented in Ref. [22] for axially symmetric flow. The latter curves assume a flat sonic profile at the throat and the wall profile is a straight

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

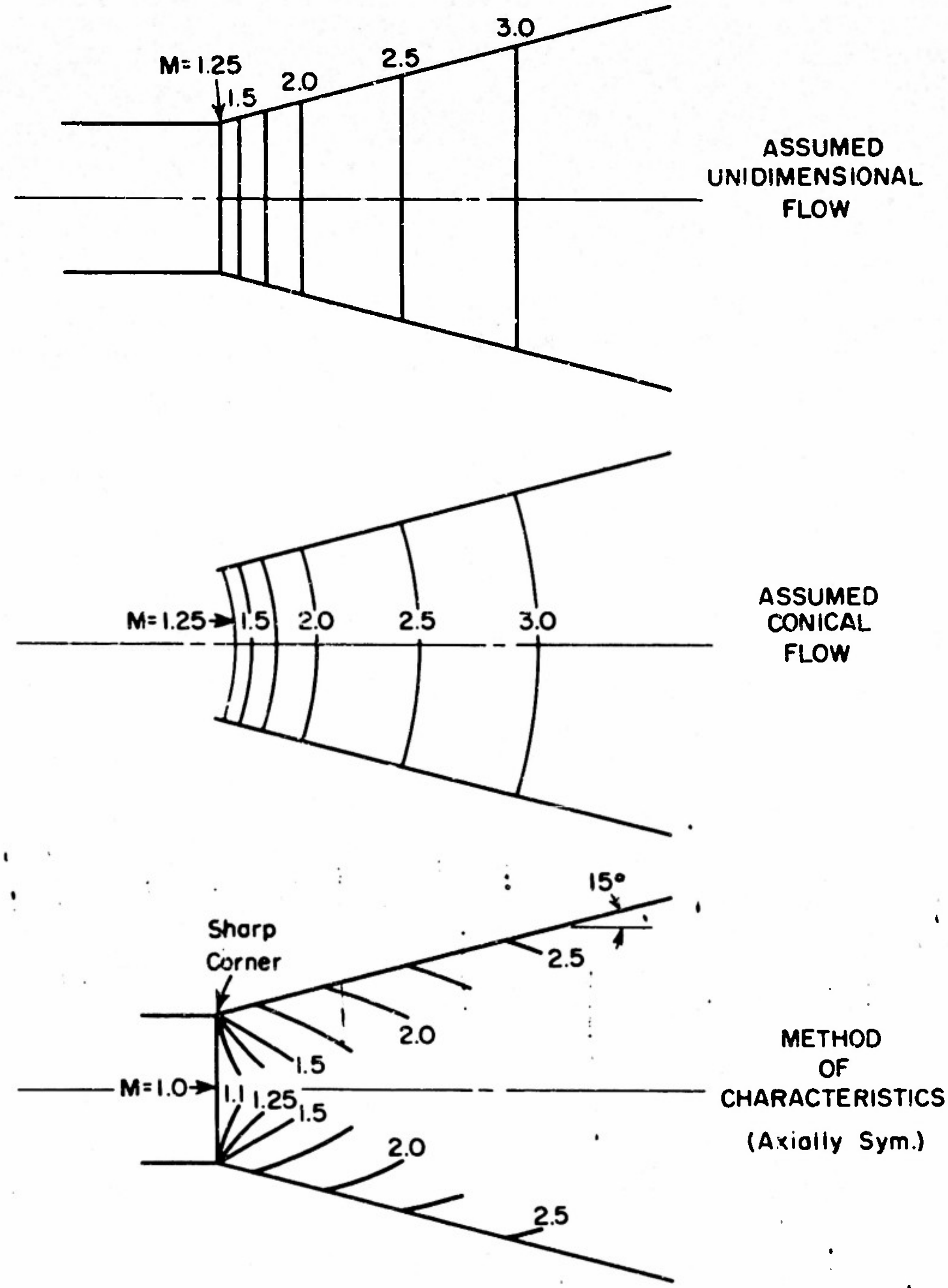


Fig. 13.3-6 COMPARISON OF MACH NUMBER PROFILES IN A CONICAL DIVERGENCE

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

line inclined 15 degrees to the axis (β = 30 degrees). A sharp corner is assumed at the throat. It is apparent that the Mach number profiles for the various methods of analysis are quite different. Although the method of characteristics is the most rigorous and should most nearly define the true flow conditions, there is some evidence that extreme care must be exercised in its application to a particular problem.

A comparison of two analyses made by the method of characteristics is shown in Fig. 13.3-7. In both cases a flat $M = 1.0$ surface and a sharp corner at the wall were assumed at the throat. In one case a perfectly straight wall was assumed having a slope of 15 degrees with the axis. The calculations were made on International Business Machine equipment described in Ref.[22]. The other solution was taken from Ref.[17] and is the upstream portion near the throat of a nozzle designed to have a uniform Mach number of 3.041 at the exit. It is clear from Fig. 13.3-7 that there is a marked difference in the Mach number profiles. Although the boundary conditions are different, the mean wall slopes are very nearly the same. There is, however, a maximum of about 3 degrees difference in wall slope at the sharp corner.

Evidently there is a slight difference in the method of starting the two solutions since the $M = 1.5$ lines (which are part of the initial expansion fan at the corner) are not identical.

The main purpose of Fig. 13.3-7 is to illustrate the large differences which can arise from seemingly small differences in wall slope and/or method of starting the solution. It should be noted, however, that the wall Mach numbers are in fair agreement to about $M = 2.0$ although the interior Mach numbers disagree.

Although the flat $M = 1.0$ profile assumption is good for wind-tunnel nozzles having long well-formed convergent sections, the characteristics solution should be started with a curved sonic profile for the shorter nozzles used in ramjets. An estimate of the shape of the sonic curve can be obtained from the preceding section on convergent nozzles.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

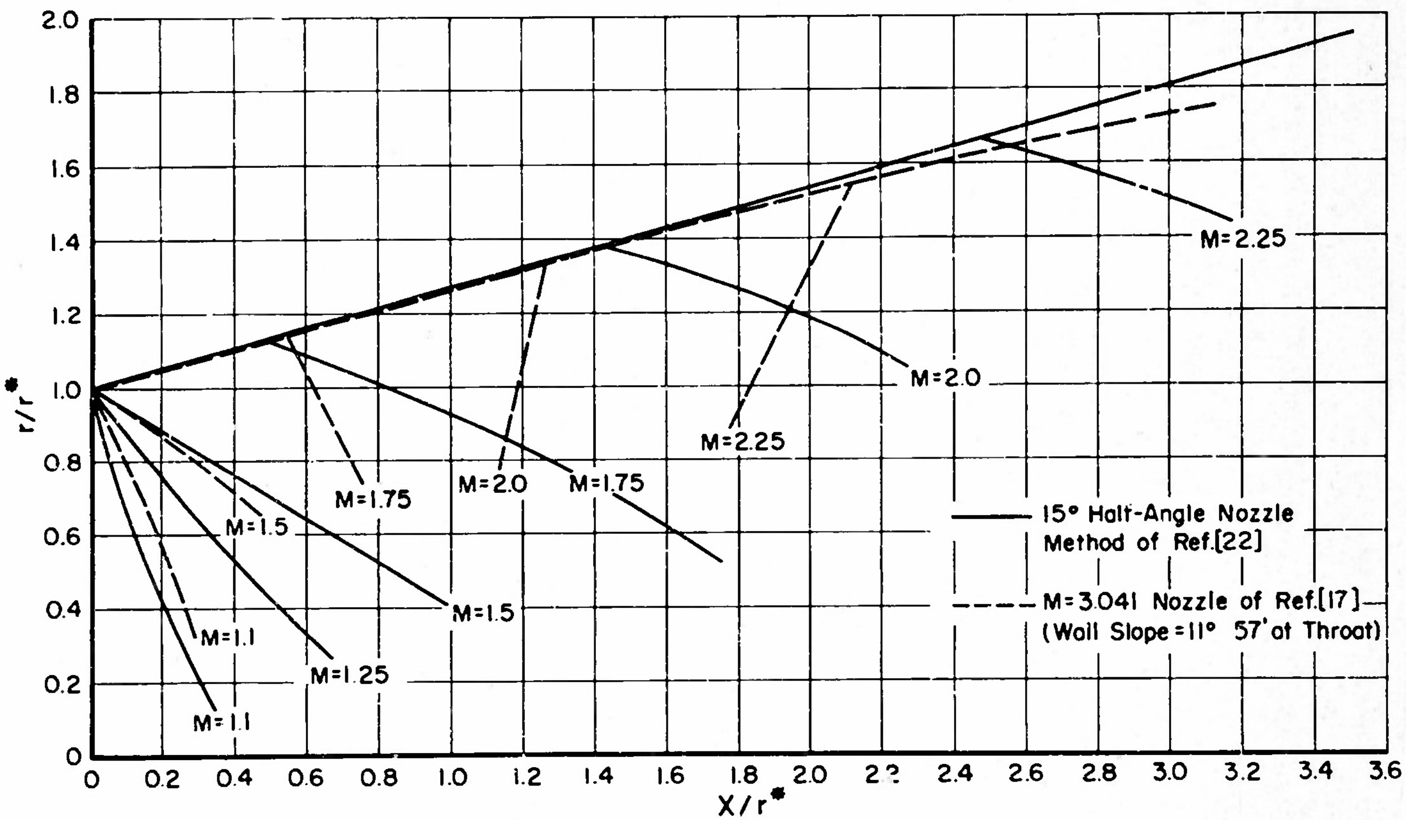


Fig. 13.3-7 COMPARISON OF MACH NUMBER PROFILES OF TWO NOZZLES HAVING SIMILAR AVERAGE WALL SLOPES NEAR THE THROAT

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

An estimate of the stream-thrust loss can be made on the basis of the characteristics method solution. Such an estimate has been made for sharp-cornered divergences with a flat-sonic profile at the throat. Solutions with curved-sonic profiles were not available. It was assumed that the stream thrust at the throat equalled the unidimensional value and that the divergent flow was isentropic. The exit stream thrust was obtained from an integration of the static pressure over the divergent area.

$$F_e = F^* + \int P dA$$

$$= F^* + P_t A^* \int P/P_t \; d(A/A^*)$$

$$\phi M_e = F_e/F^* = 1 + \frac{P_t A^*}{F^*} \int \frac{P}{P_t} \; d(A/A^*)$$

$$= 1 + \frac{1}{(f/P_t)^*} \int_{A^*}^{A} (P/P_t) \; d(A/A^*) \qquad (13.3\text{-}11)$$

It was found convenient to assume an increment of area d(A/A*) and evaluate the average P/P_t over each increment. For example, if the throat area is normalized to unity and d(A/A*) is taken as 0.05, the average P/P_t is obtained between 1.00 and 1.05, between 1.05 and 1.10, etc. By successively summing up these incremental values, and using Eq. (13.3-11), an integrated value of ϕM_e along the nozzle can be obtained. Substitution of this value in Eq. (13.3-5) gives the values of C_{f_e} as a function of area ratio of expansion.

Figures 13.3-8 and 13.3-9 show the results of such integrations for two sharp-cornered divergences, one having 15-degree and the other a 30-degree half angle. Interpolation of Fig. 13.3-8

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

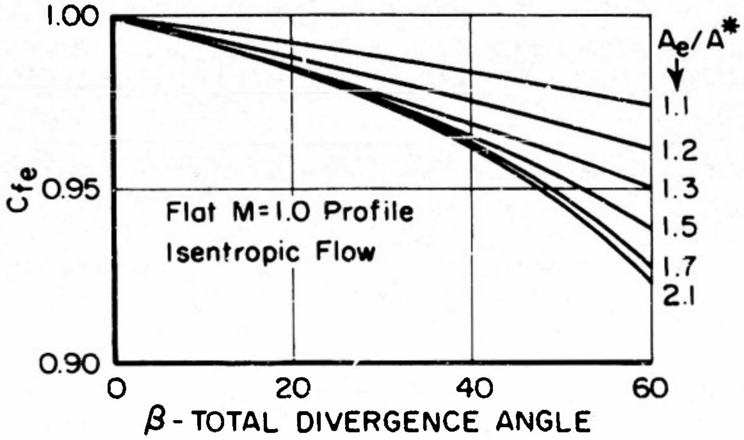


Fig. 13.3-8 EXIT STREAM THRUST COEFFICIENT FOR A SHARP-CORNERED DIVERGENCE

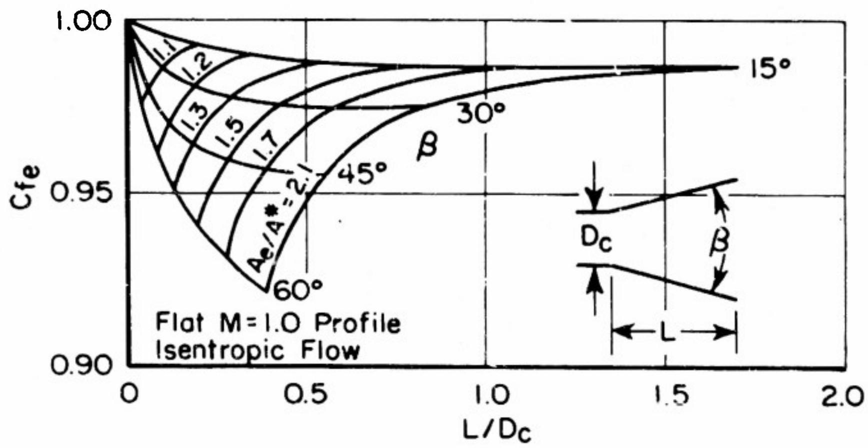


Fig. 13.3-9 EFFECT OF LENGTH ON THE STREAM THRUST COEFFICIENT FOR A SHARP-CORNERED DIVERGENCE

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

was used to obtain Fig. 13.3-9. The latter shows that there is very little thrust gain by using total-divergence angles less than about 20 degrees. Using the Talos Lot I exit divergence as an example ($L/D_c \approx 0.4$ and A_e/A_c = 1.35), it is apparent from Fig. 13.3-9 that doubling its length would produce little gain in exit-stream thrust.

It is possible to use a flow streamline as a nozzle boundary so that a given characteristics solution could supply data for several contours. In view of Fig. 13 3-7, however, the safest course is to make a characteristics solution for the particular wall contour since apparently even small wall deviations are important.

Special Divergence Forms

There are, besides the conical divergence, a number of exit-divergence forms which have been investigated.

Hengartner and Werwerka [18] describe tests on an exit form which had a 30-degree conical divergence with an exit reflex whose radius was 0.67 D_c which was parallel to the axis at the exit. Comparison with a nozzle having a 30-degree conical-divergence angle and a 25-degree conical-divergence angle showed practically no difference in exit stream-thrust coefficient.

A comparison was made at the San Diego Propulsion Laboratory of three supersonic exit nozzles having the same over-all length and inlet, throat and exit areas. Two of these had conical divergences, one 30-degree total angle and the other a 40-degree total angle. The third nozzle had a conical divergence of 36 degrees total angle followed by a reflex having a radius of curvature of about 1.0 D_c. The exit stream thrust coefficient of the reflexed nozzle was almost identical to that of the 30-degree conical divergence, the 40-degree divergence nozzle being about one per cent lower [9].

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

From these two examples it may be concluded that, for nozzle-divergence angles of about 20-30 degrees total angle, the addition of a reflexed exit causes a practically negligible increase in the exit stream-thrust coefficient over that of a conical divergence having roughly the same length and exit area ratio

Two convex-(wall slope increasing with increasing distance from the throat) divergent sections have been investigated [18]. One had a radius of curvature of 0.5 D_c and the other about 2.7 D_c. The performance of the nozzle having a short radius of curvature was almost identical to that of a sonic nozzle. The nozzle having the longer radius of curvature had an exit stream thrust coefficient of about one per cent less than that of a conical divergence of the same length.

The use of a cylindrical shroud downstream of an abrupt (β = 180 degrees total angle) expansion is presently being investigated by The Johns Hopkins University, Applied Physics Laboratory. Preliminary results have been informally reported. With no shroud the pressure on the surface surrounding the exit is near ambient even with pressure ratios in excess of the critical pressure ratios. If the shroud length is increased, with these high pressure ratios the expanding jet apparently seals against the shroud causing the pressure on the annular area at the throat to increase. The magnitude of these pressures is reported to be equivalent to a C_{f_e} of about 0.97. In these tests a well-formed inlet convergence was used. If similar results can be obtained with a simple constriction, say an orifice, it would be possible to make a very simple variable exit nozzle for use where thrust losses of this magnitude can be tolerated.

The Effects of Separation

Heretofore, unseparated flow in the divergence was discussed; the details of separation will now be considered.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C., SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

For unidimensional flow the exit-stream pressure exactly equals the pressure of the discharge region when the nozzle is operated at the nozzle-design pressure ratio (P_{t_b}/P_e). For higher nozzle-pressure ratios the flow within the nozzle divergence is identical to that at the design-pressure ratio except for the pressure and, therefore, density. There is an expansion external to the nozzle to the ambient discharge pressure. For lower pressure ratios than the design-pressure ratio there is a change in the flow pattern required which, in the unidimensional case, is a normal shock within the nozzle followed by subsonic diffusion to the ambient pressure.

The mechanism for real flows is essentially the same except for several important differences. At the design-pressure ratio the exit-stream pressure is very nearly equal to the discharge-ambient pressure for long well-formed divergences while for short divergences there will be considerable variations of pressure and Mach number across the exit plane. These variations will not change, however, as nozzle-pressure ratio is varied above the design-pressure ratio. For nozzle-pressure ratios below the design-pressure ratio, the pressure is lower than the ambient-discharge pressure at least in part of the divergence. The return to ambient-discharge pressure takes place through oblique shocks. For nozzle-pressure ratios, P_{t_b}/P_e, only slightly lower than the design-pressure ratio these oblique shocks occur outside the nozzle. As the nozzle-pressure ratio is reduced the oblique-shock system moves progressively toward the nozzle throat. Thus, for nozzle-pressure ratios in excess of that for which the oblique-shock system just enters the nozzle, the flow conditions of Mach number within the divergence are constant. Consequently, the value of C_{f_e} for a particular nozzle should be constant above a certain nozzle-pressure ratio. Since ramjets tend to operate at nozzle-pressure ratios which are relatively constant, the large practical convenience of C_{f_e} becomes clear.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

The distinct difference between separated and unseparated flow is shown in Figs. 13.3-1 and 13.3-2. The thrust advantage that results from a divergence arises from the increased pressure on the divergent area relative to the ambient (or boattail) pressure. The pressure at any particular point relative to, for example, the inlet total pressure is constant for unseparated flow.

The measured wall-static pressure profiles in the divergent section of a small exit nozzle having a forty-degree total conical divergence connected to the throat with a radius equal to $D_c/2$ is shown in Fig. 13.3-10. The unidimensional design-pressure ratio of this nozzle was about 5.0 based on the exit-area ratio. It is quite evident that the flow conditions change markedly as the nozzle-pressure ratios are reduced below about 4.0. Note that the pressure curves approach the exit ambient pressure asymptotically.

The exit stream-thrust coefficient can be expressed in terms of the static pressure on the divergent section. Burning in the divergence is neglected and C_{f_e} is based on the actual airflow. Wall friction is neglected.

$$F_e = F^* + \int PdA$$

$$W_a S_a \, \phi M_e = W_a S_a \, \phi M^* + \int PdA$$

$$\phi M_e = \left(1 + \frac{\int PdA}{F^*}\right) \frac{1}{C_d}$$

$$= \left(1 + \frac{\int (P/P_t^* \; dA/A^*)}{(f/P_t)^*}\right) \frac{1}{C_d} \qquad (13.3\text{-}12)$$

C_{f_e} may be obtained by substituting ϕM_e in Eq. (13.3-5) using ϕM_{e_i} from the exit to throat-area ratio.

CONFIDENTIAL

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C., SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

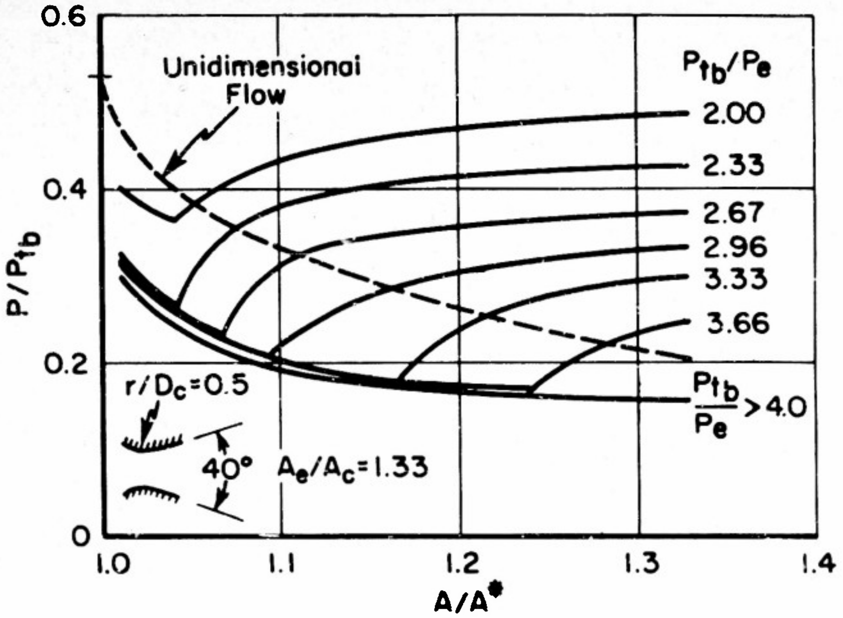


Fig. 13.3-10 VARIATION OF WALL PRESSURE RATIOS IN A PARTICULAR CONICAL DIVERGENCE

CONFIDENTIAL

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

Equation (13.3-12) indicates that the value of C_{f_e} will depend on the area under the P/P_t vs $A/A*$ curve. If the measured static pressures had equalled the values for unidimensional flow (Fig. 13.3-10) the value of C_{f_e} would have been exactly 1.00. Since, for unseparated flow, the wall pressures are almost always below the unidimensional values, C_{f_e} is less than unity under these conditions. The deviation of C_{f_e} from unity is measured by the area difference between the actual and the unidimensional pressure curves. From the shape of the curves for separated flow it is clear that, for low nozzle pressure ratios (P_{t_b}/P_e), the value of C_{f_e} will exceed unity since the area under the actual pressure curve exceeds the area under the unidimensional pressure curve. Typically, a curve of C_{f_e} versus nozzle pressure ratio, P_{t_b}/P_e, (where P_e is the pressure of the discharge region) is relatively constant for unseparated flow and gradually rises even to values above unity as nozzle-pressure ratio (P_{t_b}/P_e) is reduced. Figure 13.3-11 shows experimentally determined values of C_{f_e} calculated from data obtained in Refs. [6,10, and 23].

When the pressure on the nozzle-wall drops below the base pressure which would exist with separated flow, it will be advantageous to have separation in the divergence. The exit-stream thrust as well as the net thrust will be increased by separation.

Consider Fig. 13.3-12 which is a replot of the data from Fig. 13.3-10 for an exit pressure in the discharge region of 10 lb/in^2 absolute. The upper curve (P_{t_b} = 100) indicates under-expansion so that additional thrust could be obtained by expanding to a higher exit area (A/A*). For the P_{t_b} = 50 lb/in^2 absolute curve, (assuming 10 lb/in^2 absolute to be the base pressure with separation) slightly more thrust would have been obtained if

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

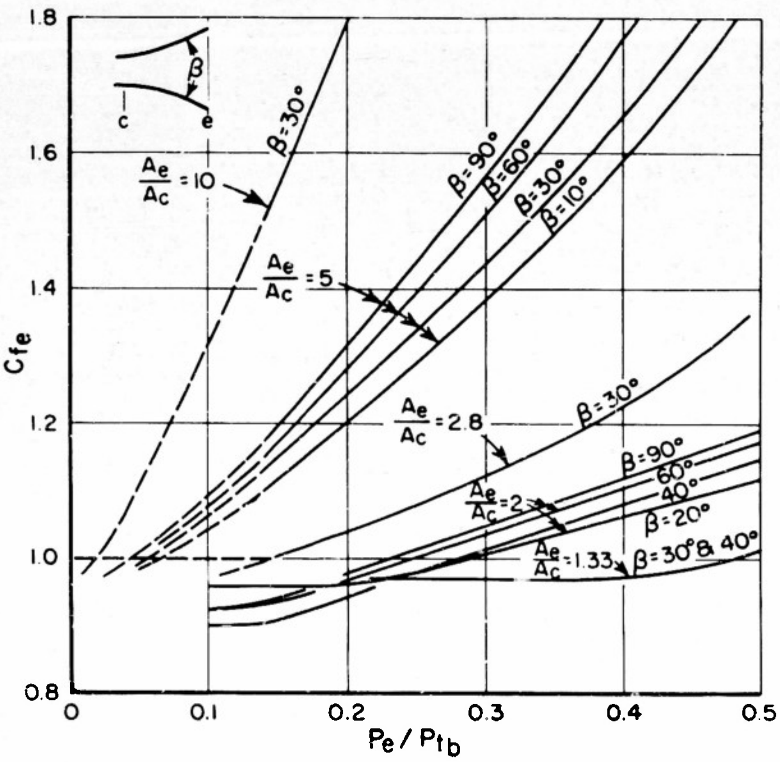


Fig. 13.3-11 EFFECT OF SEPARATION ON EXIT STREAM THRUST COEFFICIENT

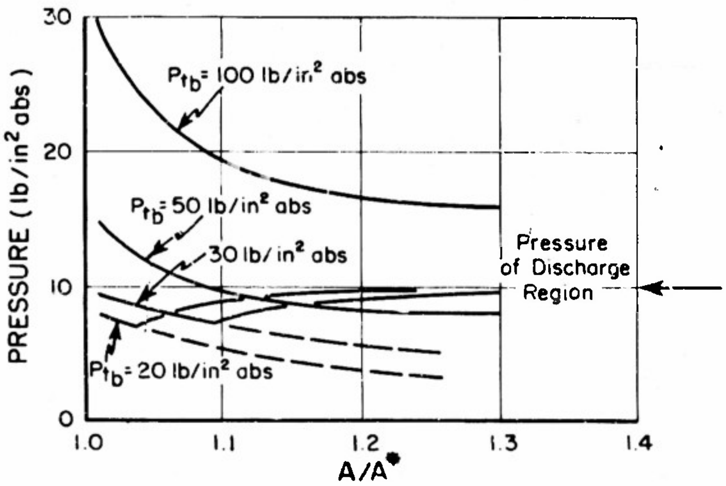


Fig. 13.3-12 VARIATION OF WALL PRESSURES IN A PARTICULAR CONICAL DIVERGENCE

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

separation occurred at an A/A* of about 1.1 since the wall pressure falls below the ambient pressure. This might be considered as equivalent to a divergence which expands to A/A* = 1.1 with a blunt exit face upon which the pressure is approximately 10 lb/in^2 absolute. For total pressures of 30 and 20 lb/in^2 absolute it would be better to have separation very near the throat, thus a sonic nozzle. It is assumed for this discussion that the base pressure for a blunt face at the exit would be very nearly equal to the base pressure resulting from separated flow in the nozzle divergence.

The nozzle pressure ratio at which the oblique-shock system first enters the nozzle divides one range of nozzle pressure ratio where C_{f_e} is constant from another range of pressure ratio wherein C_{f_e} is dependent upon the pressure ratio, exit area ratio and divergence angle.

It does not seem possible at present to define the pressure ratio at which separation first occurs at the nozzle exit. From tests reported in Refs. [18] and [21] it was found that the nozzle pressure ratio could be reduced to about one-half the design-pressure ratio before separation influenced the nozzle performance. These tests covered a range of conical divergent total angles from five to sixty degrees with an exit area ratio of about 2.1.

The pressure ratio at which separation entered the nozzle varied considerably [9] with the particular divergence used, the variation being from 50 to 70 per cent of the design pressure ratio for the conical or reflex divergences tested whose total-divergence angle ranged from 30 to 40 degrees. For two long divergences designed by the method of characteristics, separation entered the nozzle at a nozzle pressure ratio just slightly above choking, about 40 per cent of the design pressure ratio.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

Foster and Cowles found (Ref. [25]) that separation occurs even at the nozzle-design pressure ratio for total divergence angles of 60 to 80 degrees.

One of the most complete investigations of separation (Ref. [23]) found that the separation-pressure ratio, P_s/P_{t_b}, for conical divergences was a direct function of nozzle pressure ratio and varied slightly with divergence angle as long as the boundary layer was turbulent. (P_s is the wall static pressure at the point of separation.) With a laminar boundary layer, the separation pressure ratio was higher than for the turbulent boundary layer and was also dependent upon the pressure in the discharge region.

The ratio of the separation-pressure ratio (P_s/P_{t_b}) to the reciprocal of the nozzle-pressure ratio (P_e/P_{t_b}) is tabulated below for the various total conical divergence angles. P_e is the pressure of the discharge region. In the following table this ratio is reduced to the simpler ratio, P_s/P_e. Also tabulated is the ratio of the mean pressure on the separated region to the pressure of the discharge region for an exit area ratio A_e/A_c = 5.0. The tabulated values are for the turbulent boundary layer.

TABLE 13.3-1

Total Divergence Angle	P_s/P_e	P_s mean$/P_e$
10°	0.36	0.80
20°	0.44	0.86
30°	0.48	0.95
60°	0.50	0.96

The criterion (see Ref. [23]) for judging whether the boundary layer was laminar or turbulent was the length Reynolds number based upon the distance from the throat to the point of separation using values for viscosity at the inlet total temperature.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

$$R = \frac{L_s G_s}{\mu} \qquad (13.3\text{-}13$$

where G_s is the mass flow per unit area at the point of separation. The transition from dependence on discharge pressure to independence of discharge pressure occurred at length Reynolds numbers of 2×10^5 to 4×10^5. Roughening of the inlet or an increase of inlet total temperature caused by burning, produced artificial turbulence and gave the lower Reynolds number. The nozzles tested had a throat 1.10 inches in diameter, a total inlet convergence of 60 degrees, and a throat-generating radius of 1.0 D_c. Nozzle pressure ratios from two to twenty were used.

An oblique shock may accompany separation (Ref.[24]). The ratio of the nozzle area at the separation point to the throat area was found (Ref.[26]) to agree with theory if the deflection of the jet boundary at the separated point were approximately $18^\circ \pm 3^\circ$ Exit total-divergence angles from twenty to sixty degrees were used.

Foster and Cowles (Ref.[25]) indicated that an increase in the throat-generating radius improves the thrust when the flow is separated.

Since the exact point of separation and the pressures downstream of the separated point are quite difficult to predict by present theory, it seems necessary to establish the value of C_{f_e} by tests if accuracy in the separated region is desired. Particular attention should be paid to simulation of full-scale Reynolds numbers. Ideally these tests should be made with a supersonic stream external to the nozzle exit with consideration given to the external boundary layer.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

Nozzle-Pressure Ratio Effects

For most ramjets the nozzle pressure ratio will not vary widely since the flight Mach number does not usually vary appreciably. This means that the exit nozzle can operate somewhere near its design-pressure ratio and, unless separated, the exit stream-thrust coefficient will be constant for a given exit-nozzle configuration and the flow in the nozzle will be relatively independent of the exit pressure. With separation, however, the pressure on the nozzle surface downstream of the separated point is affected by the discharge pressure so that proper estimation of this value is important.

No general investigation applicable to ramjet exit nozzles has been performed to determine the exit-pressure effects resulting from a nozzle discharging into a concentric supersonic stream. Present theory does not permit a precise determination of the exit pressures, but the base pressure can be used as an approximation to determine the separation characteristics of a given nozzle and its exit stream thrust coefficient.

Since separation depends on the boundary layer on the nozzle wall, the pressure at the rim of the exit would appear to be the important pressure for determining separation.

Estimates of the base pressure can be obtained from Ref. [27] as a function of Mach number.

It has been shown (Ref. [28]) that the base pressure coefficient depends on the type of boundary layer flow, the Mach number ahead of the base, and dimensionless boundary layer thickness, δ/D where D is the body diameter.

The effect of the discharge-region pressure on separated flow is one of the nozzle parameters which needs further experimental study using an external supersonic stream.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

Effects of Burning in the Divergence

Burning in the divergent section has two principal effects: (a) The total temperature or S_a is raised, and (b) the Mach numbers are altered in the divergent region. Because of the very small residence time of most divergence sections, the amount of burning should be small. There are, however, conditions in a homogeneous reaction which could result in appreciable burning even in a relatively short time.

As Eq. (13.3-5) shows, the rise in S_a would multiply directly into the equation for C_{f_e}. There is also a loss of total pressure and a Mach number change resulting from burning which depends on the Mach number at which the burning takes place. Since the rate of burning and the Mach numbers throughout the divergence are, in general, not known and are almost impossible to define, no general solution for burning in a divergence is practical. As a first approximation, one could make an assumption as to the rate of burning and by means of equations similar to Eqs. (13.2-23) and (13.2-24), the total pressure loss from burning and Mach numbers at successive stations in the divergence could be found. From this calculation the change in C_{f_e} could be obtained.

The total pressure loss as a result of burning in a supersonic divergence has been calculated [30] for a particular rate of heat addition, i.e., the percentage rise in total temperature is proportional to the percentage increase in area. The exit Mach number can be calculated from $(\mathring{Pm}/P_t)_e$ from the following equation assuming unidimensional flow and $\gamma = 9/7$:

$$(\mathring{Pm}/P_t)_e = \frac{0.5162\sqrt{T_{t_e}/T_t^*}}{(P_{t_e}/P_t^*)(A_e/A^*)} \qquad (13.3\text{-}14)$$

CONFIDENTIAL

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

With the exit Mach number known, the ratio of the exit stream thrust with burning in the exit divergence to the stream thrust with no burning can be calculated for the same airflow and nozzle geometry.

$$\frac{F_e}{F_{e_i}} = \frac{W_a S_{a_e} \phi M_e}{W_a S_a * \phi M_{e_i}} = \sqrt{\frac{T_{t_e}}{T_t *}} \; \frac{\phi M_e}{\phi M_{e_i}} \qquad (13.3\text{-}15)$$

This ratio is plotted in Figs. 13.3-13 and 13.3-14. The increase in stream thrust is less than the square root of the temperature ratio because of the exit Mach number reduction. If the burning had taken place in the combustion chambe the exit stream thrust would have increased by the square root of the temperature ratio for the same airflow. Consequently it is better to burn in the combustor rather than in the nozzle. Figure 13.3-15 shows the ratio of the exit stream thrust with burning in the exit divergence to the exit stream thrust with burning ahead of the nozzle for the same airflow, total temperature ratio, and nozzle geometry.

Scale Effects

Practically all of the data for exit nozzle performance have been obtained with small-scale models. Recent comparisons of the C_{f_e} and the wall pressure profile for the Talos Lot I exit nozzle have shown that there is relatively good agreement within the ability to measure the full-scale burning performance. Comparison of cold flow tests of a small-scale model (approximately 2-inch throat diameter) and burning tests of the full-scale Lot I Talos exit nozzle (22-inch throat diameter) shows that the wall pressure ratios in the divergence and the nozzle pressure ratio for the beginning of separation effects were in fairly good

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

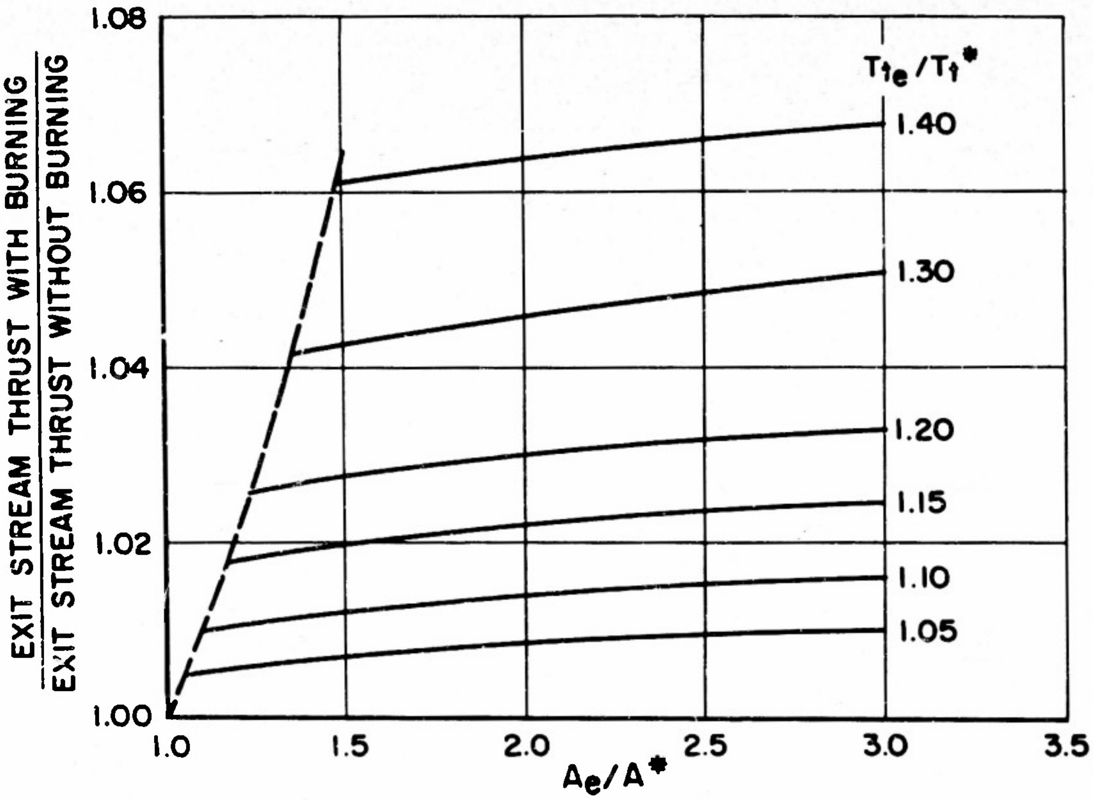


Fig. 13.3-13 EXIT STREAM THRUST INCREASE AS A RESULT OF BURNING IN THE DIVERGENCE

The same airflow and nozzle geometry total-pressure loss as used in Ref.[29].

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

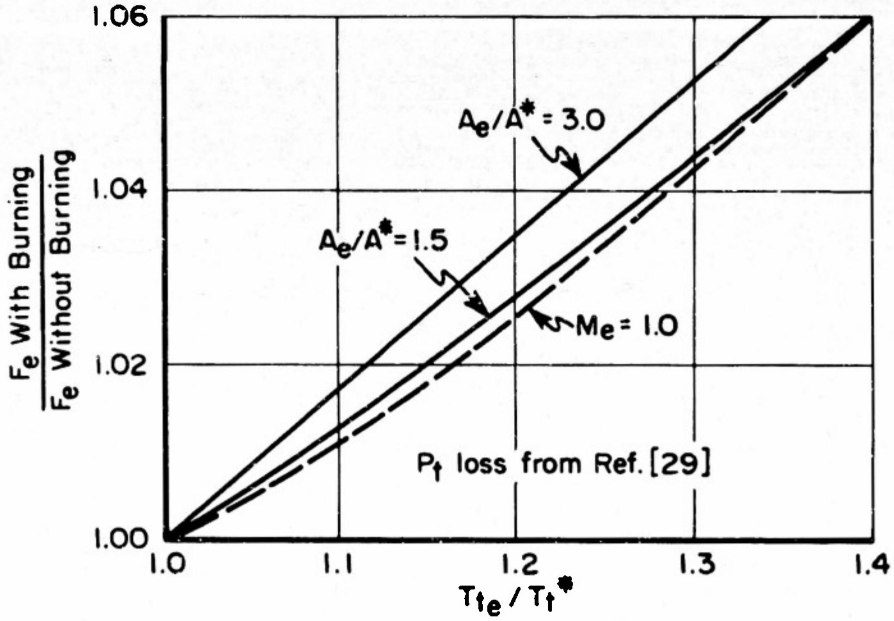


Fig. 13.3-14 EXIT STREAM THRUST INCREASE AS A RESULT OF BURNING IN THE DIVERGENCE

The airflow and nozzle geometry are the same as Ref. [29].

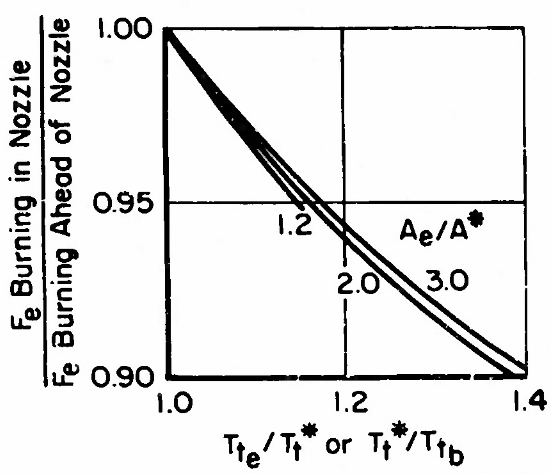


Fig. 13.3-15 RATIO OF EXIT STREAM THRUST WITH BURNING IN NOZZLE TO EXIT STREAM THRUST WITH BURNING AHEAD OF NOZZLE

For the same airflow and nozzle geometry P_t loss in divergence from Ref. [29].

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

agreement. The throat Reynolds number of the full-scale nozzle was duplicated in the model tests which, from the preceding discussions, has been shown to be important.

In view of the difficulty of measuring the burning performance of full-scale nozzles and of supplying the required airflow, a comparison of several sizes at various Reynolds numbers would be desirable to definitely establish scale effects.

Variable Exit Nozzles

There has been very little development of variable exit nozzles for ramjets because of the additional mechanical complexity and the relatively small performance gains available with present missile requirements. Consequently there is not much information of practical value on the use of variable exit nozzles. Although the preceding discussions apply directly to fixed exit nozzles the general ideas, at least, apply to a particular position of a variable exit nozzle.

The only practical means of controlling missile speed with a fixed exit nozzle (aside from increasing the drag) is by control of the exit temperature by varying the fuel flow. If combustion or diffuser efficiency decreases as the fuel flow is changed, the missile performance will suffer. Use of a variable exit nozzle will permit the designer a freedom of choice as to the best operating conditions for a particular point on the trajectory. One might estimate the performance at various points on the trajectory by using several different fixed exit nozzles that would closely approximate the variable exit nozzle to be used. Although the data of this chapter will be of some help, it is probable that tests of the particular variable design will be necessary, particularly if accuracy is required. Tests of two variable nozzles are reported in Ref. [9] which employed axially movable center plugs. The performance of these nozzles was approximately equal to a fixed nozzle of the same length and equivalent area ratio.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

13.4 OVER-ALL DESIGN CONSIDERATIONS

Important Parameters

The basic nozzle geometry has been shown to greatly affect the over-all missile optimization so that such parameters as the constriction ratio, A_c/A_b, the expansion ratio, A_e/A_c, and the length to throat diameter ratio, L/D_c, are quite dependent upon the missile application.

The burner area, A_b, is usually made as large a percentage of the maximum area as structural limitations will allow. The constriction ratio is determined mainly from the C_t requirements (Eq. 13.1-1). The remaining parameters requiring definition are A_e/A_c, L/D_c, the division of length between the subsonic and supersonic section and the wall contours.

The most important criterion for definition of these parameters is that the integrated pressure over the projected area at the exit between the throat area and the maximum missile area be a maximum.

For those trajectories in which the missile cruises for a relatively long time at an "on design" condition, the application of the above criterion will be relatively straightforward because only one nozzle pressure ratio or, at most, a very narrow band of nozzle pressure ratios will need to be investigated. Missiles whose trajectories call for fairly large Mach number variations will require optimization at several nozzle-pressure ratios and selection of the best compromise. The effort spent will, of course, depend on the missile-efficiency requirements.

For high-impulse powerplants, in which thrust is relatively more important than the utmost efficiency, the following general comments will be sufficient to obtain a fairly good compromise design. For high efficiency, low-impulse powerplants, further recommended refinements are listed in a following section.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

General Guides to Design

1. Use the maximum available area for A_e (since ΦM_{e_i} increases with A_e/A_c) except where it appears that there will be considerable overexpansion. In the latter case consider using a long boattail which may result in higher pressures on the boattailed area than would exist on the separated nozzle wall. It will rarely, if ever, be found desirable to expand to an area greater than the maximum area otherwise required.
2. The length of the inlet convergence should be as short as possible to reduce friction effects but not so short as to require a radius of wall curvature at the throat less than about 0.5 D_c. Shorter radii can be used and the resulting higher Mach numbers at the wall counteracted by means of a reflex in the diverging section. An inlet profile composed of arcs or a conical section followed by an arc at the throat will probably be most practical.
3. The length of the diverging section will be determined principally by the acceptable thrust loss resulting from divergence angle and the available length. The best divergent section considering ease of manufacturing, thrust loss and length appears to be a conical section with about 20 to 30 degrees divergence angle. Where length is critical, the effect of using a higher divergence angle can be estimated from the figures given in this chapter.
4. In most instances it will be found advantageous to favor the exit divergence with regard to length using 60 to 70 per cent of the total length for the divergence.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

5. Neglect of burning in the divergence and the resulting P_t loss will result in conservative thrust estimates.
6. To a first approximation, the discharge coefficient can be accounted for by considering the value of A_c used in the equations to be the product of the geometrical area and C_d. Since the values of C_{f_c} and C_{f_e} are defined in terms of the actual airflow no further correction to these factors is required.
7. Total pressure losses in the nozzle inlet will probably not exceed one or two per cent from friction and about five per cent from burning. For relatively long tailpipes the burning loss should be even less.
8. Determine the cooling requirements from Chapter 12.

Recommended Refinements

Where exit-nozzle efficiency is of great importance the following refinements to the general design principles are recommended:

1. Use a slightly better inlet section. A profile roughly elliptical with a length of about 0.5 D_c will probably be, for fairly high constriction ratios, a good compromise between friction and distortion of the Mach number profile. Use a long radius of wall curvature at the throat.
2. Using the Mach number profile at the throat, determine the wall Mach number and pressures by the method of characteristics of several contours which appear to be near optimum. Estimate the gains from

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

using a total divergence angle from 15 to 20 degrees as well as the slight advantages of a reflex at the exit.

3. Consider the optimum nozzle length from the standpoint of weight versus fuel specific impulse.
4. Estimate any possible gains resulting from burning in the exit nozzle by reason of the residence time of the gases.
5. Consider the total pressure loss from friction and burning in the nozzle inlet. Use the best available integrated values of $(f/P_t)_c$ and $(P\dot{m}/P_t)_c$ or their equivalent in the equations for C_t and I_f.
6. Where there is no single design point and appreciable time or fuel is spent at an off-design condition consider the gains from a variable exit nozzle.
7. Using the above detailed information for the particular nozzle proposal, make optimizations over the region of area at the exit between the throat area and the maximum missile area.
8. Conduct tests to verify, at least comparatively, the nozzle designs using an external supersonic stream. The full-scale Reynolds numbers should be duplicated as nearly as possible.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

NOMENCLATURE

		Unit
A	area	ft^2
C_d	discharge coefficient	
C_f	stream thrust coefficient	
C_t	thrust coefficient	
D	diameter	
F	stream thrust = mV + PA	
f	stream thrust per unit area	lb
$f_{app.}$	apparent laminar friction factor	
G	mass velocity = W/A	
I_f	fuel specific impulse	
L	axial length	
M	Mach number	
m	mass flow = W/g	lb/sec
$\mathring{m}$	mass flow function of Mach number	
N	ratio of thrust loss to discharge loss	
P	pressure	
q	dynamic pressure = $\rho V^2/2g$ = $P \gamma M^2/2$	
R_c	Reynolds number based on throat diameter	
R_L	Reynolds number based on axial length	
r	radius of curvature of wall	
S_a	air specific impulse	
T	temperature	°Rankine
V	velocity	
W	weight flow	
x	distance along the axis	

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C., SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

α	angle between the wall and the axis at the throat for conical convergence	
β	total angle between walls of a conical divergence. Total angle between outside boundaries of conical flow in a divergence	
γ	ratio of specific heats of a gas	
Δ	incremental value	
δ	boundary layer thickness	
δ^*	displacement thickness	
θ	momentum thickness	
η	combustion efficiency	
μ	viscosity	
π	Pi	
ρ	density	slugs/sec
ϕ_β	axial velocity defect for a conical flow	
M	ratio of stream thrust at M to stream thrust at M - 1.0	

Subscripts

am	ambient
b	Station b just ahead of nozzle contraction
bt	boattail
c	Station c at nozzle throat
d	discharge (as in C_d)
e	Station e at nozzle divergence exit
f	stream thrust (as in C_f)
i	"ideal" or isentropic unidimensional value
o	free stream
t	total or stagnation condition
t	thrust (as in C_t)

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

REFERENCES

1. Maxwell, N. E. and Shutts, W. H., Aerodynamic Effects of Boattailing on a Body of Revolution (Confidential), Applied Physics Laboratory, The Johns Hopkins University, CM-645, 1951.

2. Shapiro, H. H. and Smith, R. D., Friction Coefficients in the Inlet Length of Smooth Round Tubes, NACA TN 1785, 1948.

3. Mangler, W., Compressible Boundary Layers on Bodies of Revolution, Consolidated Vultee Aircraft Corp., German Trans., GT 204, 1946.

4. Weil, H., Effect of Pressure Gradient on Stability and Skin Friction in Laminar Boundary Layer in Compressible Fluids, J. Aero. Sci., 18, 311, (1951).

5. Van Driest, E. R., Turbulent Boundary Layer in Compressible Fluids, J. Aero. Sci., 18, 145, (1951).

6. Oswatitsch, K. and Rothstein, W., Flow Pattern in a Converging-Diverging Nozzle, NACA TM 1215, 1949.

7. Sauer, R., General Characteristics of the Flow Through Nozzles at Near Critical Speeds, Consolidated Vultee Aircraft Corp., German Trans., GT 407, NACA TM 1147, 1947.

8. Snow, R. M., Variational Methods in the Theory of Gas Flow through Nozzles, Applied Physics Laboratory, The Johns Hopkins University, CM-535, 1949.

9. Streiff, M. L., Small Scale Nozzle Tests, Consolidated Vultee Aircraft Corp; Applied Physics Laboratory, The Johns Hopkins University, CF-2025, unpublished.

10. Palmer, W. and Bennett, W., Small Scale Supersonic Exit Nozzle Test Results, Consolidated Vultee Aircraft Corp., TB-71, 1950.

11. Cunningham, R. G., Orifice Meters with Supercritical Compressible Flow, (Translation) American Soc. Mech. Eng., 73, 625, (1951).

12. Rouse, H. and Abul-Tetouh, A. H., Characteristics of Irrotational Flow Through Axially Symmetric Orifices, J. Appl. Mech., 17, 421, (1950).

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

13. Grey, R. E. and Wilsted, H. D., Performance of Conical Jet Nozzles in Terms of Flow and Velocity Coefficients, NACA TN 1757, 1948.

14. Kennedy, E. C., Sonheim, D. W., and Barnes, A. M., New Mach Numbers for Ramjet Flow Analysis (γ = 7/5 and 9/7), Consolidated Vultee Aircraft Corp., Memo 50; Applied Physics Laboratory, The Johns Hopkins University, CF-1798, 1952.

15. Dailey, C. and Wood, F., Computation Curves for Compressible Fluid Problems (γ = 1.2 to 1.4), John Wiley, New York, 1949.

16. Edelman, G. M., The Design Development and Testing of the Two-Dimensional Sharp Cornered Supersonic Nozzles, Massachusetts Institute of Technology, Meteor Report No. 22, 1948.

17. Clippenger, R. F., Supersonic Axially Symmetric Nozzles, Ballistics Research Laboratory, Report 794, 1951.

18. Hengartner, H. and Werwerka, The Most Favorable Expansion Angle of Laval Nozzles, Consolidated Vultee Aircraft Corp., German Trans., GT 802, 1949.

19. Bennett, W. and Palmer, W., Results of San Diego Tests to Determine the Effect of Nonuniform Temperature Profiles on Nozzle Performance, Consolidated Vultee Aircraft Corp., TB-62, 1950.

20. Fraser, R., Connor, J., and Coulter, M., The Measurement of the Reaction of Convergent-Divergent and Inverted Nozzles at High Pressure, British Imperial College of Science, F72/204, 1946.

21. Hengartner, H., The Influence of the Expansion Ratio on the Characteristics of a Laval Nozzle, Consolidated Vultee Aircraft Corp., German Trans., GT 667.

22. Denton, J., Application of the Method of Characteristics for Axially Symmetric Supersonic Flow to the Design Study of Talos Supersonic Nozzles, Consolidated Vultee Aircraft Corp., TB-126.

23. Kuhns, R., Flow Separation in Overexpanded Supersonic Nozzles, Consolidated Vultee Aircraft Corp., RT-124, 1952.

24. McKenney, J., An Investigation of Flow Separation in a Two-Dimensional Transparent Nozzle, California Institute of Technology, JPL Progress Report 20-129, 1951.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C. SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW

25. Foster, C. R. and Cowles, F. B., Experimental Study of the Divergence-Angle Effect in Rocket Motor Exhaust Nozzles, California Institute of Technology, JPL Progress Report 20-134, 1952.

26. Foster, C. R. and Cowles, F. B., Experimental Study of the Gas Flow Separation in Overexpanded Exhaust Nozzles for Rocket Motors, California Institute of Technology, JPL Progress Report 4-103, 1949.

27. Bonney, E., Engineering Supersonic Aerodynamics, McGraw Hill Book Co., New York, 1950.

28. Chapman, D., Base Pressure at Supersonic Velocity (Confidential), California Institute of Technology, JPL Report 4-40, 1948.

29. Hearth, D. P. and Perchonok, E., Analysis of Heat Addition in a Convergent-Divergent Nozzle, NACA TN 2938, 1953.

THIS DOCUMENT CONTAINS INFORMATION AFFECTING THE NATIONAL DEFENSE OF THE UNITED STATES WITHIN THE MEANING OF THE ESPIONAGE LAWS, TITLE 18, U.S.C., SECTIONS 793 AND 794. THE TRANSMISSION OR THE REVELATION OF ITS CONTENTS IN ANY MANNER TO AN UNAUTHORIZED PERSON IS PROHIBITED BY LAW.

UNCLASSIFIED

UNCLASSIFIED